Name Key Date

LESSON 3.1 **Study Guide**
For use with pages 152–159

GOAL **Solve systems of linear equations.**

Vocabulary

A **system of two linear equations** in two variables x and y, also called a linear system, consists of two equations that can be written in the following form:
$Ax + By = C$ and $Dx + Ey = F$

A **solution of a system** of linear equations in two variables is an ordered pair (x, y) that satisfies each equation.

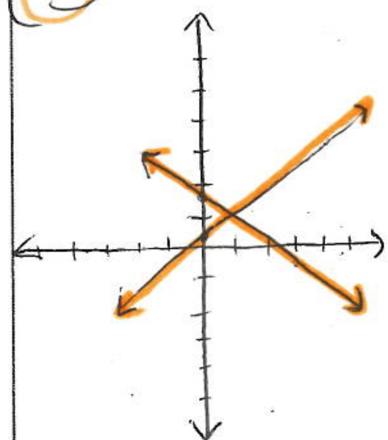
A system that has at least one solution is **consistent.**

If a system has no solution, the system is **inconsistent.** The graph of the system is a pair of parallel lines.

A consistent system that has exactly one solution point is **independent.**

A consistent system that has infinitely many solutions is **dependent.** The graph of the system is lines that coincide.

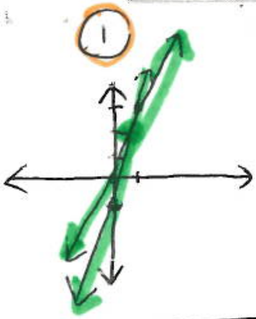
3



EXAMPLE 1

Solve a system graphically

1



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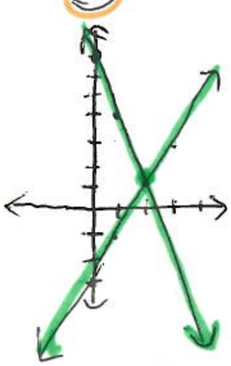


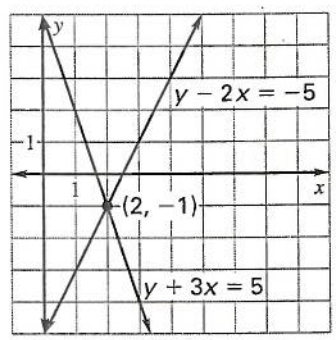
Graph the linear system and estimate the solution. Then check the solution algebraically.

$y + 3x = 5$ **Equation 1**

$y - 2x = -5$ **Equation 2**

Solution

Begin by graphing both equations, as shown at the right. From the graph, the lines appear to intersect at $(2, -1)$. The solution can be checked algebraically:



Equation 1	**Equation 2**
$y + 3x = 5$	$y - 2x = -5$
$(-1) + 3(2) \stackrel{?}{=} 5$	$-1 - 2(2) \stackrel{?}{=} -5$
$-1 + 6 \stackrel{?}{=} 5$	$-1 - 4 \stackrel{?}{=} -5$
$5 = 5$ ✓	$-5 = -5$ ✓

The solution is $(2, -1)$.

Exercises for Example 1

Graph the linear system and estimate the solution. Then check the solution algebraically.

1. $y = 4x - 1$
$y = 3x$
$\boxed{(1, 3)}$

2. $3x + y = 7$ $y = -3x + 7$
$y = 2x - 3$
$(2, 1)$

3. $2x + 3y = 5$ $y = -\frac{2}{3}x + \frac{5}{3}$
$3x - 4y = -1$ $y = \frac{3}{4}x + \frac{1}{4}$
$\boxed{(1, 1)}$

Name Key Date

LESSON 3.2

Study Guide

For use with pages 160–167

GOAL **Solve linear systems algebraically.**

Vocabulary

To use the **substitution method,** Step 1 is to *solve* one of the equations for one of its variables. Step 2 is to *substitute* the expression from Step 1 into the other equation and solve for the other variable. Step 3 is to *substitute* the value from Step 2 into the revised equation from Step 1 and solve.

To use the **elimination method,** Step 1 is to *multiply* one or both of the equations by a constant to obtain coefficients that differ only in sign for one of the variables. Step 2 is to *add* the revised equations from Step 1 and solve for the remaining variable. Step 3 is to *substitute* the value obtained in Step 2 into either of the original equations and solve for the other variable.

EXAMPLE 1 Use the substitution method

Solve the system using the substitution method.

$6x + 3y = 12$ **Equation 1**

$3x + y = 5$ **Equation 2**

Solution

STEP 1 **Solve** Equation 2 for y.

$y = 5 - 3x$

STEP 2 **Substitute** the expression for y into Equation 1 and solve for x.

$6x + 3(5 - 3x) = 12$ Substitute $5 - 3x$ for y.

$x = 1$ Solve for x.

STEP 3 **Substitute** the value of x into Equation 2 and solve for y.

$3(1) + y = 5$ Substitute 1 for x.

$y = 2$ Solve for y.

The solution is $(1, 2)$.

Exercises for Example 1

Solve the system using the substitution method.

1. $2x + y = 4$ ($y = -2x + 4$)
$3x - 5y = 6$
$3x - 5(-2x + 4) = 6$
$3x + 10x - 20 = 6$
$13x = 26$
$x = 2$
$y = -2(2) + 4$
$= 0$
$\boxed{(2, 0)}$

2. $3x + 6y = 3$
$x - 2y = 5$
$x = 2y + 5$
$3(2y + 5) + 6y = 3$
$6y + 15 + 6y = 3$
$12y = -12$
$y = -1$
$x = 2(-1) + 5$
$= 3$
$\boxed{(3, -1)}$

3. $2x - y = 6$ ($y = 2x - 6$)
$-3x + 2y = -8$
$-3x + 2(2x - 6) = -8$
$-3x + 4x - 12 = -8$
$x = 4$
$y = 2(4) - 6$
$= 2$
$\boxed{(4, 2)}$

Name Key Date

LESSON 3.3

Study Guide

For use with pages 168–173

GOAL **Graph systems of linear inequalities.**

> **Vocabulary**
>
> The following is an example of a **system of linear inequalities** in two variables: $x + y \leq 6$ and $2x - y > 6$.
>
> A **solution of a system of inequalities** is an ordered pair that is a solution of each inequality in the system.
>
> The **graph of a system of inequalities** is the graph of all solutions of the system.

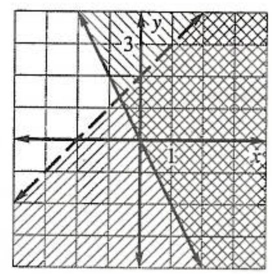
EXAMPLE 1 Graph a system of two inequalities

Graph the system of inequalities.

$y < x + 2$ **Inequality 1**

$y \geq -2x$ **Inequality 2**

Solution



STEP 1 **Graph** each inequality in the system. Shade $y < x + 2$ and shade $y \geq -2x$.

STEP 2 **Identify** the region that is common to both graphs. It is the region that is shaded darkest.

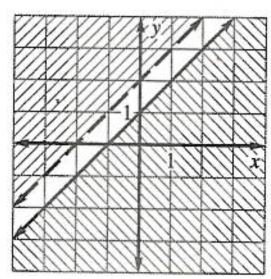
EXAMPLE 2 Graph a system with no solution

Graph the system of inequalities.

$y > x + 2$ **Inequality 1**

$y \leq x + 1$ **Inequality 2**

Solution



STEP 1 **Graph** each inequality in the system. Shade $y > x + 2$ and shade $y \leq x + 1$.

STEP 2 **Identify** the region that is common to both graphs. There is no common region shaded by both inequalities. So, the system has no solution.

Name Key Date

LESSON 3.4

Study Guide

For use with pages 177–185

GOAL **Solve systems of equations in three variables.**

Vocabulary

A **linear equation in three variables** x, y, and z is an equation of the form $ax + by + cz = d$ where a, b, and c are not all zero.

An example of a **system of three linear equations** in three variables:

$x + 2y + z = 3$ Equation 1

$2x + y + z = 4$ Equation 2

$x - y - z = 2$ Equation 3

A **solution of a system with three variables** is an **ordered triple** (x, y, z) whose coordinates make each equation true.

EXAMPLE 1 Use the elimination method

Solve the system.

$2x + 3y - z = 13$ **Equation 1**

$3x + y - 3z = 11$ **Equation 2**

$x - y + z = 3$ **Equation 3**

STEP 1 **Rewrite** the system as a linear system in two variables.

$$\begin{array}{r} 2x + 3y - z = 13 \\ 3x - 3y + 3z = 9 \\ \hline 5x \qquad + 2z = 22 \end{array}$$

Add 3 times the third equation to the first equation. New Equation 1

$$\begin{array}{r} 3x + y - 3z = 11 \\ x - y + z = 3 \\ \hline 4x \qquad - 2z = 14 \end{array}$$

Add the second and third equations. New Equation 2

STEP 2 **Solve** the new linear system for both of its variables.

$$\begin{array}{r} 5x + 2z = 22 \\ 4x - 2z = 14 \\ \hline 9x \qquad = 36 \end{array}$$

Add new Equation 1 and new Equation 2.

$x = 4$ Solve for x.

$z = 1$ Substitute into new Equation 1 or 2 to find z.

STEP 3 **Substitute** $x = 4$ and $z = 1$ into an original equation and solve for y.

$x - y + z = 3$ Write original Equation 3.

$4 - y + 1 = 3$ Substitute 4 for x and 1 for z.

$y = 2$ Solve for y.

The solution is $x = 4$, $y = 2$, and $z = 1$ or the ordered triple $(4, 2, 1)$.

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LESSON 3.5

Study Guide

For use with pages 187–194

GOAL **Perform operations with matrices.**

Vocabulary

A **matrix** is a rectangular arrangement of numbers in rows and columns.

The **dimensions** of a matrix with m rows and n columns are $m \times n$.

The numbers in a matrix are its **elements.**

Equal matrices have the same dimensions and the elements in corresponding positions are equal.

To perform **scalar multiplication,** you multiply a matrix by a real number (called a **scalar**) by multiplying each element in the matrix by the scalar.

EXAMPLE 1

Add and subtract matrices

Perform the indicated operation, if possible.

a. $\begin{bmatrix} -9 & 0 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 7 \\ 10 & -2 \end{bmatrix} = \begin{bmatrix} -9+1 & 0+7 \\ -1+10 & 5+(-2) \end{bmatrix} = \begin{bmatrix} -8 & 7 \\ 9 & 3 \end{bmatrix}$

b. $\begin{bmatrix} -8 & -5 \\ 7 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} -8-1 & 5-2 \\ 7-8 & 9-(-2) \end{bmatrix} = \begin{bmatrix} -9 & -7 \\ -1 & 11 \end{bmatrix}$

Exercises for Example 1

Perform the indicated operation, if possible.

1. $\begin{bmatrix} 3 & -4 \\ 0 & 6 \end{bmatrix} + \begin{bmatrix} -7 & 5 \\ 10 & -2 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 10 & 4 \end{bmatrix}$

2. $\begin{bmatrix} -6 & -4 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} +1 & -3 \\ +5 & +4 \end{bmatrix} = \begin{bmatrix} -5 & -7 \\ 2 & 11 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -1 & 3 \\ 8 & -9 & 6 \end{bmatrix} + \begin{bmatrix} -3 & 1 & -2 \\ 4 & 6 & 7 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 & 1 \\ 12 & -3 & 13 \end{bmatrix}$

4. $\begin{bmatrix} 0 & -5 & 8 \\ 3 & -3 & 6 \\ 4 & 7 & -2 \end{bmatrix} + \begin{bmatrix} +4 & -1 & +1 \\ -9 & +5 & -3 \\ -5 & -8 & -1 \end{bmatrix}$

$\begin{bmatrix} 4 & -6 & 9 \\ -6 & 2 & 3 \\ -1 & -1 & -3 \end{bmatrix}$

EXAMPLE 2

Multiply a matrix by a scalar

Perform the indicated operation.

$$-3\begin{bmatrix} 0 & 3 \\ -2 & 5 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} -3(0) & -3(3) \\ -3(-2) & -3(5) \\ -3(1) & -3(4) \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 6 & -15 \\ -3 & -12 \end{bmatrix}$$

Name Key Date

LESSON 3.6

Study Guide

For use with pages 195–202

GOAL **Multiply matrices.**

EXAMPLE 1 Find the product of two matrices

Find AB if $A = \begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix}$.

Because the number of columns in A (two) equals the number of rows in B (two), the product AB is defined and is a 2×2 matrix.

STEP 1 **Multiply** the numbers in the first row of A by the numbers in the first column of B, add the products, and put the result in the first row, first column of AB.

$$\begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 3(4) + 6(-3) & \\ & \end{bmatrix}$$

STEP 2 **Multiply** the numbers in the first row of A by the numbers in the second column of B, add the products, and put the result in the first row, second column of AB.

$$\begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 3(4) + 6(-3) & 3(2) + 6(8) \\ & \end{bmatrix}$$

STEP 3 **Multiply** the numbers in the second row of A by the numbers in the first column of B, add the products, and put the result in the second row, first column of AB.

$$\begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 3(4) + 6(-3) & 3(2) + 6(8) \\ 7(4) + (-1)(-3) & \end{bmatrix}$$

STEP 4 **Multiply** the numbers in the second row of A by the numbers in the second column of B, add the products, and put the result in the second row, second column of AB.

$$\begin{bmatrix} 3 & 6 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 3(4) + 6(-3) & 3(2) + 6(8) \\ 7(4) + (-1)(-3) & 7(2) + (-1)(8) \end{bmatrix}$$

STEP 5 **Simplify** the product matrix.

$$\begin{bmatrix} 3(4) + 6(-3) & 3(2) + 6(8) \\ 7(4) + (-1)(-3) & 7(2) + (-1)(8) \end{bmatrix} = \begin{bmatrix} -6 & 54 \\ 31 & 6 \end{bmatrix}$$

Exercises for Example 1

Find the product. If it is not defined, state the reason.

1. $\underset{2\times 1}{\begin{bmatrix} 4 \\ -2 \end{bmatrix}}\underset{1\times 2}{\begin{bmatrix} 1 & 3 \end{bmatrix}}$

$\begin{bmatrix} 4 & 12 \\ -2 & -6 \end{bmatrix}$

2. $\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} -5 & -2 \\ 1 & 6 \end{bmatrix}$

$\begin{bmatrix} -10+3 & -4+18 \\ -20+7 & -8+42 \end{bmatrix}$

$\begin{bmatrix} -7 & 14 \\ -13 & 34 \end{bmatrix}$

3. $\begin{bmatrix} 1 & -3 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} -9 & 7 \\ -2 & -4 \end{bmatrix}$

$\begin{bmatrix} -9+6 & 7+12 \\ -45+-16 & 35+-40 \end{bmatrix}$

$\begin{bmatrix} -3 & 19 \\ -61 & -5 \end{bmatrix}$

Name Key Date

LESSON 3.8
Study Guide
For use with pages 210–217

GOAL **Solve linear systems using inverse matrices.**

Vocabulary

The $n \times n$ **identity matrix** is a matrix with 1's on the main diagonal and 0's elsewhere. If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $AI = A$ and $IA = A$.

Two $n \times n$ matrices A and B are **inverses** of each other if their product (in both orders) is the $n \times n$ identity matrix.

In the matrix equation $AX = B$, matrix A is the coefficient matrix, X is **the matrix of variables,** and B is the **matrix of constants.**

EXAMPLE 1 Solve a matrix equation

Solve the matrix equation $AX = B$ for the 2×2 matrix X.

$$\overbrace{\begin{bmatrix} 1 & 1 \\ 6 & 7 \end{bmatrix}}^{A} X = \overbrace{\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}}^{B}$$

Begin by finding the inverse of A.

$$A^{-1} = \frac{1}{7-6}\begin{bmatrix} 7 & -1 \\ -6 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ -6 & 1 \end{bmatrix}$$

To solve the equation for X, multiply both sides of the equation by A^{-1} on the left.

$$\begin{bmatrix} 7 & -1 \\ -6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 6 & 7 \end{bmatrix} X = \begin{bmatrix} 7 & -1 \\ -6 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad A^{-1}AX = A^{-1}B$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} 13 & 17 \\ -11 & -14 \end{bmatrix} \qquad IX = A^{-1}B$$

$$X = \begin{bmatrix} 13 & 17 \\ -11 & -14 \end{bmatrix} \qquad X = A^{-1}B$$

Exercises for Example 1

Solve the matrix equation.

1. $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} X = \begin{bmatrix} 6 & 4 \\ 2 & 8 \end{bmatrix}$

$X = \begin{bmatrix} 16 & 4 \\ -10 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$X = \begin{bmatrix} 3 & 5 \\ -2 & -3 \end{bmatrix}$

3. $\begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix} X = \begin{bmatrix} 0 & -1 \\ 3 & 2 \end{bmatrix}$

$X = \begin{bmatrix} -12 & -9 \\ 27 & 20 \end{bmatrix}$