Name Key Date

LESSON 3.8

Study Guide *continued*

For use with pages 210–217

EXAMPLE 2 Solve a linear system

Use an inverse matrix to solve the linear system.

$3x + 2y = 1$ **Equation 1**

$4x + y = -2$ **Equation 2**

Solution

STEP 1 Write the linear system as a matrix equation $AX = B$.

$$\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

STEP 2 Find the inverse of matrix A.

$$A^{-1} = \frac{1}{3-8}\begin{bmatrix} 1 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -0.2 & 0.4 \\ 0.8 & -0.6 \end{bmatrix}$$

STEP 3 Multiply the matrix of constants by A^{-1}on the left.

$$X = A^{-1}B = \begin{bmatrix} -0.2 & 0.4 \\ 0.8 & -0.6 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The solution of the system is $(-1, 2)$.

EXAMPLE 3 Solve a multi-step problem

Gift Shop A gift shop sells three beach packages. Package A that includes 2 towels, 1 tube of sunscreen, and 1 beach chair costs \$21. Package B that includes 3 towels and 2 beach chairs costs \$31. Package C that includes 1 tube of sunscreen and 2 beach chairs costs \$19. Find the cost of each package item.

STEP 1 Write a system of equations. Let t be the cost of a towel, s be the cost of the tube of sunscreen, and c be the cost of a beach chair.

$2t + s + c = 21$ Package A

$3t + 2c = 31$ Package B

$s + 2c = 19$ Package C

STEP 2 Rewrite the system as a matrix equation and solve $X = A^{-1}B$ with a graphing calculator.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} t \\ s \\ c \end{bmatrix} = \begin{bmatrix} 21 \\ 31 \\ 19 \end{bmatrix}$$

A towel costs \$5, sunscreen costs \$3, and a chair costs \$8.

4

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \end{bmatrix}$$

5

$$\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

6

$$\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Exercises for Examples 2 and 3

Use an inverse matrix to solve the linear system.

4. $2x - y = -4$, $x + y = 13$ $\boxed{(3, 10)}$

5. $3x - y = 1$, $-x + 2y = 3$ $\boxed{(1, 2)}$

6. $2x + y = -1$, $3x - y = -4$ $\boxed{(-1, 1)}$

7. Rework Example 3 with Package A = \$23, Package B = \$36, and Package C = \$20.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} t \\ s \\ c \end{bmatrix} = \begin{bmatrix} 23 \\ 36 \\ 20 \end{bmatrix} \rightarrow \begin{bmatrix} t \\ s \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix}$$

towels \$6
sunscreen \$2
chair \$9

Name Key Date

LESSON 3.6

Study Guide *continued*

For use with pages 195–202

4

AB

$$\begin{bmatrix} 0+24 & -4+-30 \\ 0+12 & 2+-15 \end{bmatrix}$$

AB

$$\begin{bmatrix} 24 & -34 \\ 12 & -13 \end{bmatrix}$$

EXAMPLE 2 Use matrix operations

Using the given matrices, evaluate the expression $B(A + C)$.

$$A = \begin{bmatrix} 2 & -1 \\ -6 & 4 \end{bmatrix},\ B = \begin{bmatrix} 4 & 2 \\ 1 & -5 \end{bmatrix},\ C = \begin{bmatrix} 0 & -3 \\ 4 & 5 \end{bmatrix}$$

$$B(A + C) = \begin{bmatrix} 4 & 2 \\ 1 & -5 \end{bmatrix}\left(\begin{bmatrix} 2 & -1 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 0 & -3 \\ 4 & 5 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 4 & 2 \\ 1 & -5 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 \\ -2 & 9 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 12 & -49 \end{bmatrix}$$

Exercises for Example 2

Using the given matrices, evaluate the expression.

$$\begin{bmatrix} -2 & 6 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 2 & -5 \end{bmatrix}$$

$$\begin{bmatrix} -10+12 & -2+-30 \\ 5+6 & 1+-15 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 6 \\ 1 & 3 \end{bmatrix},\ B = \begin{bmatrix} 0 & 2 \\ 4 & -5 \end{bmatrix},\ C = \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix}$$

4. $AB + C$ $\begin{bmatrix} 29 & -35 \\ 10 & -13 \end{bmatrix}$

5. $2AB$ use #4 $\begin{bmatrix} 48 & -68 \\ 24 & -26 \end{bmatrix}$

6. $A(B + C)$ $\begin{bmatrix} 2 & -32 \\ 11 & -14 \end{bmatrix}$

EXAMPLE 3 Use matrices to calculate total cost

School Supplies You and a friend are purchasing school supplies. You buy 4 binders, and 1 notepad. Your friend buys 2 binders and 3 notepads. Each binder costs \$3.50 and each notepad costs \$2. Write a supplies matrix and a cost per item matrix. Then use matrix multiplication to write a total cost matrix.

Solution

$$\begin{array}{lcc} & \multicolumn{2}{c}{\text{Supplies}} \\ & \text{Binders} & \text{Notepads} \\ \text{You} & \multicolumn{2}{c}{\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix}} \\ \text{Friend} & & \end{array} \qquad \begin{array}{lc} & \text{Cost} \\ & \text{Dollars} \\ \text{Binders} & \begin{bmatrix} 3.5 \\ 2 \end{bmatrix} \\ \text{Notepads} & \end{array}$$

The total cost can be found by multiplying the 2×2 supplies matrix by the 2×1 cost per item matrix. The product is a 2×1 total cost matrix.

$$\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3.5 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 13 \end{bmatrix}$$

The total cost of the school supplies is the sum of the rows of the total cost matrix, or \$29.

Exercise for Example 3

7. Rework Example 3 if a binder costs \$3 and a notepad costs \$2.50.

$$\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2.50 \end{bmatrix} = \begin{bmatrix} 12 + 2.50 \\ 6 + 7.50 \end{bmatrix} = \begin{bmatrix} 14.50 \\ 13.50 \end{bmatrix}$$

total #28

Name Key Date

LESSON 3.5

Study Guide *continued*

For use with pages 187–194

Exercises for Example 2

Perform the indicated operation.

5. $2\begin{bmatrix} -3 & 0 \\ -2 & 1 \end{bmatrix}$ $\begin{bmatrix} -6 & 0 \\ -4 & 2 \end{bmatrix}$

6. $4\begin{bmatrix} 1 & -7 \\ -3 & 0 \\ -1 & 2 \end{bmatrix}$ $\begin{bmatrix} 4 & -28 \\ -12 & 0 \\ -4 & 8 \end{bmatrix}$

7. $-3\begin{bmatrix} 5 & 4 & -2 \\ 0 & 3 & 1 \end{bmatrix}$ $\begin{bmatrix} -15 & -12 & 6 \\ 0 & -9 & -3 \end{bmatrix}$

EXAMPLE 3

Solve a multi-step problem

Gardening Last year you planted 12 geraniums, 36 begonias, and 20 petunias. This year you planted 10 geraniums, 40 begonias, and 32 petunias. Organize the data using matrices. Write and interpret a matrix that gives the change in the number of flowers for the two year period.

Solution

$$\begin{array}{l} \\ \text{Geraniums} \\ \text{Begonias} \\ \text{Petunias} \end{array} \begin{array}{c} \text{Last year} \\ \begin{bmatrix} 12 \\ 36 \\ 20 \end{bmatrix} \end{array} - \begin{array}{c} \text{This year} \\ \begin{bmatrix} 10 \\ 40 \\ 32 \end{bmatrix} \end{array} = \begin{array}{c} \text{Change} \\ \begin{bmatrix} 2 \\ -4 \\ -12 \end{bmatrix} \end{array}$$

The difference of the two matrices represents the change in the number of flowers.

Exercise for Example 3

8. Rework Example 3 to write and interpret a matrix giving the average number of flowers for the two year period.

EXAMPLE 4

Solve a matrix equation

Solve the matrix equation for *x* and *y*.

$$\begin{bmatrix} 2x & 4 \\ -9 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 3y & -1 \end{bmatrix}$$

Equate corresponding elements and solve the two resulting equations.

$2x = 8$ $\qquad$ $-9 = 3y$

$x = 4$ $\qquad$ $-3 = y$

Exercises for Example 4

Solve the matrix equation for *x* and *y*.

9. $\begin{bmatrix} 10 & -3y \\ 6 & 13 \end{bmatrix} = \begin{bmatrix} 10 & -15 \\ 6x & 13 \end{bmatrix}$

$-3y = -15$ $\qquad$ $6 = 6x$

$\boxed{y = 5}$ $\qquad$ $\boxed{1 = x}$

10. $\begin{bmatrix} 12 & 3 \\ 6y & 5 \end{bmatrix} = \begin{bmatrix} -4x & 3 \\ 24 & 5 \end{bmatrix}$

$12 = -4x$ $\qquad$ $6y = 24$

$\boxed{-3 = x}$ $\qquad$ $\boxed{y = 4}$

Name Key Date

LESSON 3.4

Study Guide *continued*

For use with pages 177–185

EXAMPLE 2 Solve a three-variable system with no solution

Solve the system. $2x - 2y + 2z = 9$ **Equation 1**

$x - y + z = 5$ **Equation 2**

$3x + y + 2z = 4$ **Equation 3**

When you multiply the second equation by -2 and add the result to the first equation, you obtain a false equation.

$$\begin{aligned} 2x - 2y + 2z &= 9 \\ -2x + 2y - 2z &= -10 \\ 0 &= -1 \end{aligned}$$ New Equation 1

Because you obtain a false equation, the original system has no solution.

EXAMPLE 3 Solve a three-variable system with many solutions

Solve the system. $x + y + z = 3$ **Equation 1**

$x + y - z = 3$ **Equation 2**

$3x + 3y + z = 9$ **Equation 3**

STEP 1 Rewrite the system as a linear system in two variables.

$$\begin{aligned} x + y + z &= 3 \\ x + y - z &= 3 \\ \hline 2x + 2y &= 6 \end{aligned}$$

Add the first equation to the second.

New Equation 1

$$\begin{aligned} x + y - z &= 3 \\ 3x + 3y + z &= 9 \\ \hline 4x + 4y &= 12 \end{aligned}$$

Add the second equation to the third.

New Equation 2

1) $3x + 3z = 9$
$-2x - 3z = -8$
$x = 1$

$2(1) + y + 2 = 5$
$y + 4 = 5$
$y = 1$

$\boxed{(1, 1, 2)}$

STEP 2 Solve the new linear system for both of its variables.

$$\begin{aligned} -4x - 4y &= -12 \\ 4x + 4y &= 12 \\ \hline 0 &= 0 \end{aligned}$$

Add -2 times new Equation 1 and new Equation 2.

Because you obtain the identity $0 = 0$, the system has infinitely many solutions.

STEP 3 Describe the solution. Divide new Equation 1 by 2 to get $x + y = 3$, or $y = -x + 3$. Substituting this into the original equation produces $z = 0$. Any ordered triple of the form $(x, -x + 3, 0)$ is a solution of the system.

Exercises for Examples 1, 2, and 3

Solve the system.

1. $3x + 3z = 9$
$2x + \cancel{y} + z = 5$
$x - \cancel{y} + 2z = 4$
$x + \cancel{y} + z = 4$
$2x + 3z = 8$
$2(1) + 3z = 8$
$3z = 6$
$z = 2$

2. $3x + 6y + 3z = 21$
$x - y + z = 4$
$3x + 6y + 3z = 9$
$0 = -12$
No solution

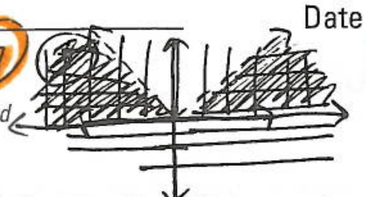
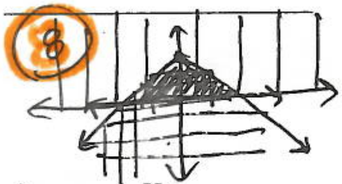
3. $x + y = 2$
$2x + 2y = 4$
$x + y + \cancel{z} = 2$
$x + y - \cancel{z} = 2$
$2x + 2y + \cancel{z} = 4$
$3x + 3y = 6$
$x + y = 2$
Infinitely many solut

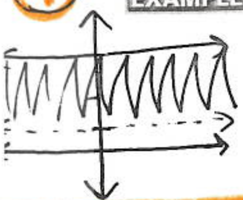
Name Key Date

LESSON 3.3

Study Guide continued

For use with pages 168–173

7  8 

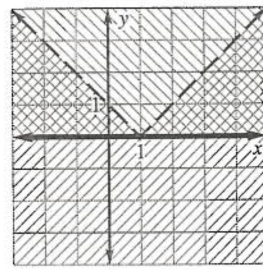
1 

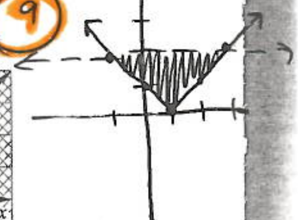
EXAMPLE 3 Graph a system with an absolute value inequality

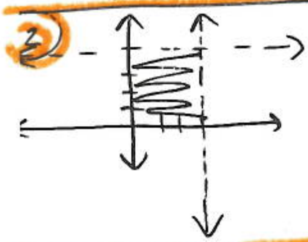
Graph the system of inequalities.

$y \geq 0$ **Inequality 1**

$y < |x - 1|$ **Inequality 2**



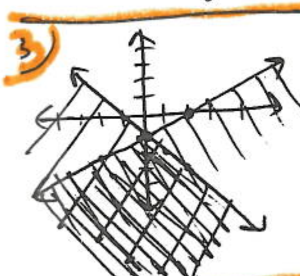
9 

2 

STEP 1 Graph each inequality in the system. Shade $y < |x - 1|$ and shade $y \geq 0$.

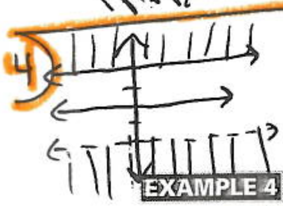
STEP 2 Identify the region that is common to both graphs. It is the region that is shaded darkest.

Exercises for Examples 1, 2, and 3

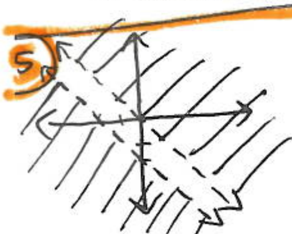
3 

Graph the system of inequalities.

1. $y > 1$ dotted
   $y \leq 3$ solid
2. $x < 3$ dotted
   $y < 4$ dotted
3. $y \leq -x - 1$ solid
   $y \leq \frac{1}{2}x - 1$ solid
4. $y \geq 2$ solid
   $y < -2$ dotted
   NO sol.
5. $y > |-x|$ dotted
   $y < |-x| - 2$ dotted
   NO sol.
6. $y \geq x + 1$ solid
   $y \leq x - 1$ solid
   NO sol.
7. $y < |x|$ dotted
   $y \geq 0$ solid
8. $y \leq -|x| + 2$ solid
   $y \geq 0$ solid
9. $y \geq |x - 1|$ solid
   $y < 2$ dotted

4 

EXAMPLE 4 Graph a system of three or more inequalities

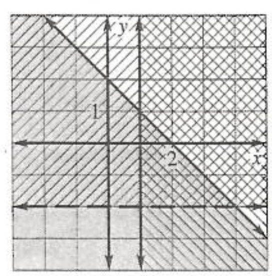
5 

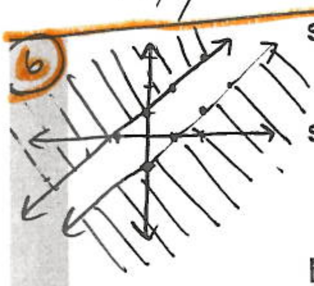
Graph the system of inequalities.

$y \leq -x + 2$ **Inequality 1**

$x \geq 1$ **Inequality 2**

$y > -2$ **Inequality 3**



6 

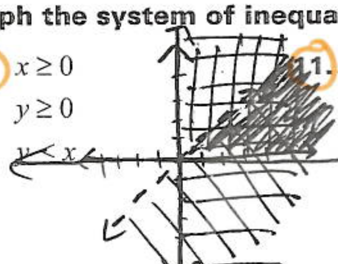
STEP 1 Graph each inequality in the system. Shade $y \leq -x + 2$, shade $x \geq 1$, and shade $y > -2$.

STEP 2 Identify the region that is common to all the graphs. The solution of the system is the region that is shaded darkest.

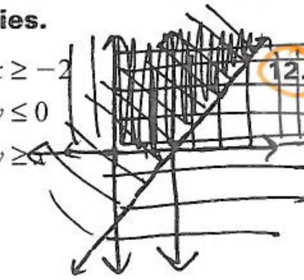
Exercises for Example 4

Graph the system of inequalities.

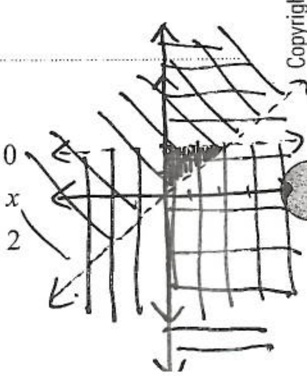
10. $x \geq 0$
    $y \geq 0$
    $y < x$



11. $x \geq -2$
    $y \leq 0$
    $y \geq x$



12. $x \geq 0$
    $y > x$
    $y < 2$



Name Key Date ______

LESSON 3.2

Study Guide *continued*

For use with pages 160–167

EXAMPLE 2 Use the elimination method

Solve the system using the elimination method.

$x + 5y = 13$ **Equation 1**

$-4x - 7y = -13$ **Equation 2**

STEP 1 Multiply Equation 1 by 4 so that the coefficients of x differ only in sign.

$x + 5y = 13 \xrightarrow{\times 4} 4x + 20y = 52$

$-4x - 7y = -13 \qquad -4x - 7y = -13$

STEP 2 Add the revised equations and solve for y.

$13y = 39$

$y = 3$

STEP 3 Substitute the value of y into Equation 1 and solve for x.

$x + 5(3) = 13$ Substitute 3 for y in Equation 1.

$x = -2$ Solve for x.

The solution is $(-2, 3)$.

EXAMPLE 3 Solve linear systems with many or no solutions

a. Solve: $9x - 3y = 6$, $3x - y = 2$

b. Solve: $x - 2y = 5$, $4x - 8y = 3$

a. $y = 3x - 2$ Solve Equation 2 for y.

$9x - 3(3x - 2) = 6$ Substitute $3x - 2$ for y.

$6 = 6$ Simplify.

Because the equation $6 = 6$ is always true, there are infinitely many solutions.

b. $-4x + 8y = -20$ Multiply Equation 1 by -4.

$4x - 8y = 3$ Add Equation 2.

$0 = -17$

Because the equation $0 = -17$ is never true, there is no solution.

Exercises for Examples 2 and 3

Solve the system using the elimination method.

$\boxed{(-1, 1)}$

4. $14x + 4y = -10$
$3x - 4y = -7$
$17x = -17$
$x = -1$

$-3 - 4y = -7$
$-4y = -4$
$y = 1$

5. $-4x + 6y = -6$
$4x - 5y = 9$
$y = 3$

$4x - 15 = 9$
$4x = 24$
$x = 6$

$\boxed{(6, 3)}$

6. $5x - 6y = 4$
$4x + 6y = 14$
$9x = 18$
$x = 2$

$10 - 6y = 4$
$-6y = -6$
$y = 1$

$\boxed{(2, 1)}$

Solve the linear system by any algebraic method.

7. $2(3x - y = 5)$
$6x - 2y = 10$
$6x - 2y = 10$

$\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$

ERROR SINGULAR

$\boxed{\text{Infinitely many solutions}}$

8. $4x + 8y = 5 \cdot 20$
$4x - 8y = -3$

$\begin{bmatrix} 1 & -2 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

$\boxed{\text{No solution}}$

9. $3x + 6y = 21$
$-3x - 6y = -7$

$\begin{bmatrix} 1 & 2 \\ -3 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -7 \end{bmatrix}$

$\boxed{\text{No solution}}$

Name Key   Date

LESSON 3.1

Study Guide *continued*

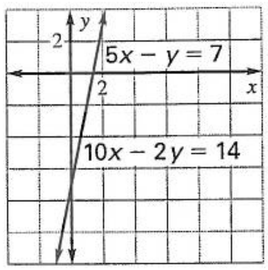
For use with pages 152–159

EXAMPLE 2 Classify a system of two linear equations

Solve each system. Then classify each system as *consistent and independent, consistent and dependent,* or *inconsistent.*

a. $10x - 2y = 14$ Equation 1

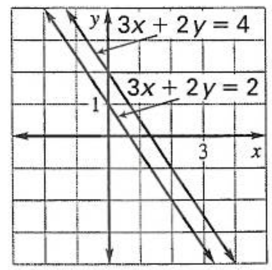
$5x - y = 7$ Equation 2



The graphs of the equations are the same line. The system is consistent and dependent.

b. $3x + 2y = 2$ Equation 1

$3x + 2y = 4$ Equation 2



The graphs of the equations are parallel lines. The system is inconsistent.

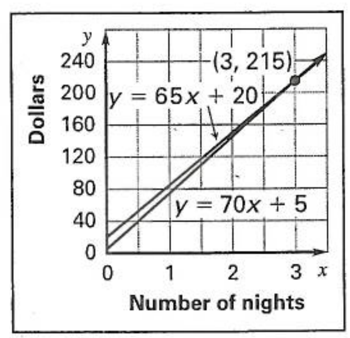
EXAMPLE 3 Writing and using a linear system

Resort A charges \$70 per night, plus a one-time surcharge of \$5. Resort B charges \$65 per night, plus a one-time surcharge of \$20. After how many nights will the total cost of the two options be the same?

$y = 70 \cdot x + 5$ Equation 1 (Resort A)

$y = 65 \cdot x + 20$ Equation 2 (Resort B)

To solve the system, graph the equations. The lines appear to intersect at (3, 215). So, after 3 nights the total cost of the two options is the same.



4

$-2x = x + 3$

$-3x = 3$

$x = -1$

$y = -2(-1)$

$= 2$

$\boxed{(-1, 2)}$

consistent + independent

Exercises for Examples 2 and 3

Solve the system. Then classify the system as *consistent and independent, consistent and dependent,* or *inconsistent.*

4. $y = -2x$
$y = x + 3$

5. $y - 3x = 2$ no sol.
$y - 3x = 3$
inconsistent

6. $x + y = 6$
$-2x - 2y = -12$
infinitely many
Consistent + dependent

7. In Example 3, suppose Resort A charges \$70 per night, plus a one-time surcharge of \$10 and Resort B charges \$60 per night, plus a one-time surcharge of \$30. After how many nights will the total cost of the two options be the same?

A → $70x + 10$

B → $60x + 30$

7

$70x + 10 = 60x + 30$

$10x = 20$

$x = \boxed{2 \text{ days}}$