Name Key Date

LESSON 4.5
Study Guide *continued*
For use with pages 266–271

LESSON 4.5

EXAMPLE 3 Solve a quadratic equation

Solve $2x^2 - 3 = 87$.

$2x^2 - 3 = 87$	Write original equation.
$2x^2 = 90$	Add 3 to each side.
$x^2 = 45$	Divide each side by 2.
$x = \pm\sqrt{45}$	Take square roots of each side.
$x = \pm\sqrt{9} \cdot \sqrt{5}$	Product property
$x = \pm 3\sqrt{5}$	Simplify.

EXAMPLE 4 Finding solutions of a quadratic equation

Find the solutions of $\frac{1}{2}(w - 2)^2 + 1 = 4$.

$\frac{1}{2}(w - 2)^2 + 1 = 4$	Write original equation.
$(w - 2)^2 = 6$	Multiply each side by 2. Subtract 2 from each side.
$w - 2 = \pm\sqrt{6}$	Take square roots of each side.
$w = 2 \pm \sqrt{6}$	Add 2 to each side.

The solutions are $2 + \sqrt{6}$ and $2 - \sqrt{6}$.

Exercises for Examples 3 and 4

Solve the equation.

7. $-9d^2 = -405$

$d^2 = 45$

$\boxed{d = \pm 3\sqrt{5}}$

8. $11y^2 + 3 = 36$

$11y^2 = 33$

$y^2 = 3$

$\boxed{y = \pm\sqrt{3}}$

9. $\frac{1}{5}j^2 + 4 = 12$

$(5)\frac{1}{5}j^2 = 8(5)$

$j^2 = 40$

$\boxed{j = \pm 2\sqrt{10}}$

EXAMPLE 5 Model a dropped object with a quadratic function

Dropping an Object How long does it takes for an object to hit the ground when dropped from a height of 72 feet?

Solution

$h = -16t^2 + h_0$	Write height function.
$0 = -16t^2 + 72$	Substitute 0 for h and 72 for h_0.
$4.5 = t^2$	Subtract 72 from each side. Divide each side by -16.
$\pm\sqrt{4.5} = t$	Take square roots of each side.
$\pm 2.1 \approx t$	Use a calculator.

Reject the negative solution because time must be positive. The object will fall for about 2.1 seconds before it hits the ground.

10

$h = -16t^2 + 120$

$0 = -16t^2 + 120$

$-120 = -16t^2$

$\sqrt{\frac{15}{2}} = \sqrt{t^2}$

$\pm \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}\ t$

$\pm \frac{\sqrt{30}}{2} = t$

Exercise for Example 5

10. Rework Example 5 where the object is dropped from a height of 120 feet.

$+\frac{\sqrt{30}}{2}$ ≈ 2.74 seconds

Name Key Date

LESSON 4.6

Study Guide

For use with pages 275–282

GOAL **Perform operations with complex numbers.**

Vocabulary

The **imaginary unit** i is defined as $i = \sqrt{-1}$.

A **complex number** written in standard form is a number $a + bi$ where a and b are real numbers. If $b \neq 0$, then $a + bi$ is an **imaginary number.**

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates.**

Every complex number corresponds to a point in the **complex plane.** The complex plane has a horizontal axis called the real axis and a vertical axis called the imaginary axis.

The **absolute value of a complex** number $z = a + bi$, denoted $|z|$, is a nonnegative real number defined as $|z| = \sqrt{a^2 + b^2}$.

EXAMPLE 1

Solve a quadratic equation

Solve $3x^2 - 1 = -16$.

Solution

$3x^2 - 1 = -16$	Write original equation.
$3x^2 = -15$	Add 1 to each side.
$x^2 = -5$	Divide each side by 3.
$x = \pm\sqrt{-5}$	Take square roots of each side.
$x = \pm i\sqrt{5}$	Write in terms of i.

The solutions are $i\sqrt{5}$ and $-i\sqrt{5}$.

EXAMPLE 2

Add and subtract complex numbers

Write the expression $9 - (10 + 2i) - 5i$ as a complex number in standard form.

Solution

$9 - (10 + 2i) - 5i = [9 - 10 - 2i] - 5i$	Definition of complex subtraction
$= (-1 - 2i) - 5i$	Simplify.
$= -1 - (2 + 5)i$	Definition of complex addition
$= -1 - 7i$	Write in standard form.

LESSON 4.6

Name Key Date

LESSON 4.6 **Study Guide** *continued*
For use with pages 275–282

6

$-24i + 8i^2$ *
-8

Exercises for Examples 1 and 2

Solve the equation.

1. $7x^2 - 13 = -20$ $7x^2 = -7$ $x^2 = -1$ $\boxed{x = \pm i}$
2. $x^2 + 14 = 2$ $x^2 = -12$ $\boxed{x = \pm 2i\sqrt{3}}$
3. $4x^2 - 5 = -77$ $4x^2 = -72$ $x^2 = -18$ $\boxed{x = \pm 3i\sqrt{2}}$

7

$-12 + 6i + 20i - 10i^2$ $+10$ *

Write the expression as a complex number in standard form.

4. $(-11 + 3i) + (4 - 6i)$ $\boxed{-7 - 3i}$
5. $15 + (9 + 4i) - 7i$ $\boxed{6 - 11i}$

EXAMPLE 3 **Multiply and divide complex numbers**

9

$\frac{(1+3i)}{2i} \cdot \frac{2i}{2i}$

$\frac{2i + 6i^2}{4i^2}$ *

$\frac{-6 + 2i}{-4}$

$\frac{-6}{-4} + \frac{2i}{-4}$

Write each expression as a complex number in standard form.

a. $(-8 - 3i)(2 + 4i) = -16 - 32i - 6i - 12i^2$ Multiply using FOIL.
$= -16 - 38i - 12(-1)$ Simplify. Use $i^2 = -1$.
$= -16 - 38i + 12$ Simplify.
$= -4 - 38i$ Write in standard form.

b. $\frac{5 - 2i}{3 + 8i} = \frac{5 - 2i}{3 + 8i} \cdot \frac{3 - 8i}{3 - 8i}$ Multiply by $\frac{3 - 8i}{3 - 8i}$.
$= \frac{15 - 40i - 6i - 16i^2}{9 - 24i + 24i - 64i^2}$ Multiply using FOIL.
$= \frac{31 - 46i}{73} = \frac{31}{73} - \frac{46}{73}i$ Use $i^2 = -1$. Write in standard from.

10

$\frac{2i}{3+i} \cdot \frac{(3-i)}{3-i}$

$\frac{6i - 2i^2}{9 - i^2}$ *

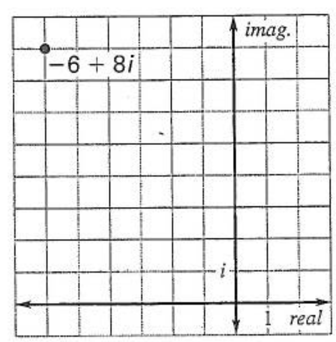
$\frac{2 + 6i}{10} \rightarrow \frac{2}{10} + \frac{6i}{10}$

EXAMPLE 4 **Plot a complex number and find its absolute value**

a. Find the absolute value of $-6 + 8i$.

$|-6 + 8i| = \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64}$
$= \sqrt{100} = 10$

b. To plot $-6 + 8i$ start at the origin, move 6 units to the left, and then move 8 units up.



11

$\frac{(4+2i)}{1-i} \cdot \frac{(1+i)}{1+i}$

$\frac{4 + 4i + 2i + 2i^2}{1 - i^2}$ *

$\frac{2 + 6i}{2}$

$\frac{2}{2} + \frac{6}{2}i$

Exercises for Examples 3 and 4

Write the product or quotient in standard form.

6. $-8i(3 - i)$ $-8 - 24i$
7. $(-3 + 5i)(4 - 2i)$ $\boxed{-2 + 26i}$
8. $(7 + 6i)(7 - 6i)$ $\boxed{85}$ $49 - 36i^2$ *
9. $\frac{1 + 3i}{2i}$ $\boxed{\frac{3}{2} - \frac{1}{2}i}$
10. $\frac{2i}{3 + i}$ $\boxed{\frac{1}{5} + \frac{3i}{5}}$
11. $\frac{4 + 2i}{1 - i}$ $\boxed{1 + 3i}$

Plot the complex numbers in the same complex plane and then find the absolute value of the complex number.

12. $-2i$
13. $5i$
14. $1 + 3i$
15. $2 - i$

Name Key Date

LESSON 4.7

Study Guide

For use with pages 283–291

GOAL **Solve quadratic equations by completing the square.**

> **Vocabulary**
>
> To **complete the square** for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.
>
> When you complete the square, $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$.

EXAMPLE 1 Make a perfect square trinomial

Find the value of c that makes $x^2 - 10x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

Solution

STEP 1 **Find** half the coefficient of x. $\quad \frac{-10}{2} = -5$

STEP 2 Square the result of Step 1. $\quad (-5)^2 = 25$

STEP 3 **Replace** c with the result of Step 2. $\quad x^2 - 10x + 25$

The trinomial $x^2 - 10x + c$ is a perfect square when $c = 25$.
So, $x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$.

Exercises for Example 1

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

$\left(\frac{12}{2}\right)^2$ $(6)^2$ **1.** $x^2 + 12x + \cancel{c}\ 36$ $\quad (x+6)^2$

$\left(\frac{-18}{2}\right)^2$ $(-9)^2$ **2.** $x^2 - 18x + \cancel{c}\ 81$ $\quad (x-9)^2$

$\left(\frac{-40}{2}\right)^2$ $(-20)^2$ **3.** $x^2 - 40x + \cancel{c}\ 400$ $\quad (x-20)^2$

EXAMPLE 2 Solve $ax^2 + bx + c = 0$ when $a = 1$

Solve $x^2 - 16x + 8 = 0$ by completing the square.

Solution

$x^2 - 16x + 8 = 0$	Write original equation.
$x^2 - 16x = -8$	Write left side in the form $x^2 + bx$.
$x^2 - 16x + 64 = -8 + 64$	Add $\left(\frac{-16}{2}\right)^2 = 64$ to each side.
$(x - 8)^2 = 56$	Write left side as a binomial squared.
$x - 8 = \pm\sqrt{56}$	Take square roots of each side.
$x = 8 \pm \sqrt{56}$	Solve for x.
$x = 8 \pm 2\sqrt{14}$	Simplify: $\sqrt{56} = \sqrt{4} \cdot \sqrt{14} = 2\sqrt{14}$

The solutions are $8 + 2\sqrt{14}$ and $8 - 2\sqrt{14}$.

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LESSON 4.7 **Study Guide** *continued*
For use with pages 283–291

EXAMPLE 3 Solve $ax^2 + bx + c = 0$ when $a \neq 1$

Solve $3x^2 + 6x + 15 = 0$ by completing the square.

$3x^2 + 6x + 15 = 0$	Write original equation.
$x^2 + 2x + 5 = 0$	Divide each side by the coefficient of x^2, 3.
$x^2 + 2x = -5$	Write left side in the form $x^2 + bx$.
$x^2 + 2x + 1 = -5 + 1$	Add $\left(\frac{2}{2}\right)^2 = 1^2 = 1$ to each side.
$(x + 1)^2 = -4$	Write left side as a binomial squared.
$x + 1 = \pm\sqrt{-4}$	Take square roots of each side.
$x + 1 = \pm 2i$	Write in terms of the imaginary unit i.
$x = -1 \pm 2i$	Solve for x.

The solutions are $-1 + 2i$ and $-1 - 2i$.

(7) $y=(x-6)^2+2$ $(6,2)$
(8) $y=(x-7)^2+1$ $(7,1)$
(9) $y=2(x+3)^2-5$
(10) vertex $(10,9000)$
10 units would maximize revenue

Exercises for Examples 2 and 3

Solve the equation by completing the square.

4. $x^2 - 10x + 6 = 0$ 5. $2x^2 + 16x + 8 = 0$ 6. $5x^2 - 10x + 30 = 0$

EXAMPLE 4 Find the maximum value of a quadratic function

Revenue A retailer's revenue is modeled by $R = (300 + 10x)(50 - x)$. Rewrite in vertex form to find the number of units x that maximizes the revenue R.

Solution

$R = (300 + 10x)(50 - x)$	Write original function.
$R = 15{,}000 - 300x + 500x - 10x^2$	Use FOIL.
$R = -10x^2 + 200x + 15{,}000$	Combine like terms.
$R = -10(x^2 - 20x) + 15{,}000$	Prepare to complete the square.
$R = -10\left[x^2 - 20x + \left(\frac{-20}{2}\right)^2\right] + 10\left(\frac{-20}{2}\right)^2 + 15{,}000$	Add and subtract $10\left(\frac{-20}{2}\right)$.
$R = -10(x - 10)^2 + 16{,}000$	Write a perfect square trinomial as the square of a binomial.

The vertex is (10, 16,000), so the number of units that maximizes R is 10.

Exercises for Example 4

Write the equation in vertex form and identify the vertex.

7. $y = x^2 - 12x + 38$ 8. $y = x^2 - 14x + 50$ 9. $y = 2x^2 + 12x + 13$

10. Rework Example 4 where $R = (200 + 10x)(40 - x)$.

LESSON 4.7

Name Key Date

LESSON 4.8

Study Guide

For use with pages 292–299

GOAL **Solve quadratic equations using the quadratic formula.**

Vocabulary

The **quadratic formula:** Let a, b, and c be real numbers where $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In the quadratic formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

EXAMPLE 1 Solve a quadratic equation with two real solutions

Solve $x^2 - 5x = 4$.

$x^2 - 5x = 4$	Write original equation.
$x^2 - 5x - 4 = 0$	Write in standard form.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-4)}}{2(1)}$	$a = 1, b = -5, c = -4$
$x = \dfrac{5 \pm \sqrt{41}}{2}$	Simplify.

The solutions are $x = \dfrac{5 + \sqrt{41}}{2} \approx 5.70$ and $x = \dfrac{5 - \sqrt{41}}{2} \approx -0.70$.

EXAMPLE 2 Solve a quadratic equation with one real solution

Solve $4x^2 + 10x = -10x - 25$.

$4x^2 + 10x = -10x - 25$	Write original equation.
$x^2 + 20x + 25 = 0$	Write in standard form.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$x = \dfrac{-20 \pm \sqrt{20^2 - 4(4)25}}{2(4)}$	$a = 4, b = 20, c = 25$
$x = \dfrac{-20 \pm 0}{8}$	Simplify.
$x = -\dfrac{5}{2}$	Simplify.

The solution is $-\frac{5}{2}$.

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LESSON 4.8

Study Guide *continued*

For use with pages 292–299

EXAMPLE 3 Solve a quadratic equation with imaginary solutions

Solve $x^2 - 6x = -10$.

$x^2 - 6x = -10$	Write original equation.
$x^2 - 6x + 10 = 0$	Write in standard form.
$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)10}}{2(1)}$	$a = 1, b = -6, c = 10$
$x = \frac{6 \pm \sqrt{-4}}{2}$	Simplify.
$x = \frac{6 \pm 2i}{2}$	Rewrite using the imaginary unit i.
$x = 3 \pm i$	Simplify.

The solutions are $3 + i$ and $3 - i$.

Exercises for Examples 1, 2, and 3

Use the quadratic formula to solve the equation.

1. $x^2 + 4x = 2$ $\boxed{-2+\sqrt{6}}$ $\boxed{-2-\sqrt{6}}$
2. $2x^2 - 8x = 1$ $\boxed{\frac{4+3\sqrt{2}}{2}}$ $\boxed{\frac{4-\sqrt{3}}{2}}$
3. $4x^2 + 2x = -2x - 1$ $\boxed{-\frac{1}{2}}$
4. $16x^2 - 20x = 4x - 9$ $\boxed{\frac{3}{4}}$
5. $x^2 - 4x + 5 = 0$ $\boxed{2+2i}$ $\boxed{2-2i}$
6. $x^2 - x = -7$ $\boxed{\frac{1+3i\sqrt{3}}{2}}$ $\boxed{\frac{1-3i\sqrt{3}}{2}}$

1. $A=1$, $B=4$, $C=-2$
$x = \frac{-4 \pm \sqrt{4^2-4(1)(-2)}}{2(1)} = \frac{-4 \pm \sqrt{24}}{2} = \frac{-4 \pm 2\sqrt{6}}{2}$

2. $A=2$, $B=-8$, $C=-1$
$x = \frac{8 \pm \sqrt{(-8)^2-4(2)(-1)}}{2(2)} = \frac{8 \pm \sqrt{72}}{4} = \frac{8 \pm 6\sqrt{2}}{4}$

3. $A=4$, $B=4$, $C=1$
$x = \frac{-4 \pm \sqrt{4^2-4(4)(1)}}{2(4)} = \frac{-4}{8}$

4. $A=16$, $B=-24$, $C=9$
$x = \frac{24 \pm \sqrt{(-24)^2-4(16)(9)}}{2(16)} = \frac{24 \pm 0}{32}$

5. $A=1$, $B=-4$, $C=5$
$x = \frac{4 \pm \sqrt{(-4)^2-4(1)(5)}}{2(1)} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2}$

6. $A=1$, $B=-1$, $C=7$
$x = \frac{1 \pm \sqrt{(-1)^2-4(1)(7)}}{2(1)} = \frac{1 \pm \sqrt{-27}}{2} = \frac{1 \pm 3i\sqrt{3}}{2}$

EXAMPLE 4 Use the discriminant

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

a. $x^2 + 6x + 11$ **b.** $x^2 + 6x + 9$ **c.** $x^2 + 6x + 5$

Solution

	Equation	Discriminant	Solution(s)
	$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
a.	$x^2 + 6x + 11 = 0$	$6^2 - 4(1)(11) = -8$	Two imaginary: $-3 \pm i\sqrt{2}$
b.	$x^2 + 6x + 9 = 0$	$6^2 - 4(1)(9) = 0$	One real: -3
c.	$x^2 + 6x + 5 = 0$	$6^2 - 4(1)(5) = 16$	Two real: $-5, -1$

Exercises for Example 4

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

7. $x^2 - 2x - 1$
$(-2)^2 - 4(1)(-1)$ $\boxed{8}$ $\boxed{\text{2 real}}$

8. $x^2 - 12x + 36$
$(-12)^2 - 4(1)(36)$ $\boxed{0}$ $\boxed{\text{1 real}}$

9. $x^2 + 7x + 14$
$(7)^2 - 4(1)(14)$ $\boxed{-7}$ $\boxed{\text{2 imaginary}}$

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LESSON 4.10

Study Guide *continued*

For use with pages 308–315

EXAMPLE 3 Write a quadratic function in standard form

Write a quadratic function in standard form for the parabola that passes through the points (−1, 2), (0, −2), and (1, 0).

STEP 1 Substitute the coordinates of each point into $y = ax^2 + bx + c$.

$2 = a(-1)^2 + b(-1) + c$	Substitute −1 for x and 2 for y.
$2 = a - b + c$	Equation 1
$-2 = a(0)^2 + b(0) + c$	Substitute 0 for x and −2 for y.
$-2 = c$	Equation 2
$0 = a(1)^2 + b(1) + c$	Substitute 1 for x and 0 for y.
$0 = a + b + c$	Equation 3

STEP 2 Rewrite the system of three equations in Step 1.

$4 = a - b$	Revised Equation 1: Substitute −2 for c.
$2 = a + b$	Revised Equation 3: Substitute −2 for c.

STEP 3 Solve the system of revised Equations 1 and 3. By the elimination method, $a = 3$ and $b = -1$. The solution of the system is $a = 3, b = -1$, and $c = -2$. A quadratic function for the parabola is $y = 3x^2 - x - 2$.

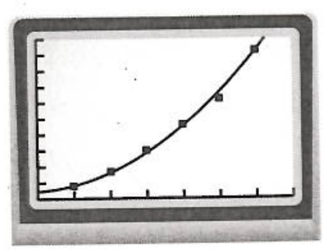
EXAMPLE 4 Find a best-fitting quadratic model for data

The table shows the population of a bacteria culture from day 5 to day 30. Use a graphing calculator to find the best-fitting quadratic model.

Day	5	10	15	20	25	30
Population	60	135	240	370	500	740

STEP 1 Enter the data into two lists of a graphing calculator.

STEP 2 Make a scatter plot of the data.



STEP 3 Use the *quadratic regression* feature to find the best-fitting quadratic model for the data, $y \approx 0.71x^2 + 2.1x + 37$.

STEP 4 Check how well the model fits the data by graphing the model and the data in the same viewing window.

Exercises for Examples 3 and 4

Write a quadratic function in standard form for the parabola that passes through the given points.

5. (−2, 0), (1, 0), (2, −4) $y = -x^2 - x + 2$

6. (−1, 0), (0, −5), (2, 3) $y = 3x^2 - 2x - 5$

7. Rework Example 4 using the (day, population) values: (4, 56), (8, 100), (12, 165), (16, 255), (20, 360), (24, 500).
$y = .71x^2 + 2.07x + 37$

LESSON 4.10

Name

Date

LESSON 4.10

Study Guide

For use with pages 308–315

GOAL **Write quadratic functions and models.**

> **Vocabulary**
> The model given by quadratic regression is called the **best-fitting quadratic model.**

EXAMPLE 1 Write a quadratic function in vertex form

Write a quadratic function whose graph has vertex (−2, 3) and passes through the point (−1, 1).

Solution

$y = a(x - h)^2 + k$ Vertex form

$y = a(x + 2)^2 + 3$ Substitute −2 for h and 3 for k.

Use the point (−1, 1) to find a.

$1 = a(-1 + 2)^2 + 3$

$1 = a + 3$

$a = -2$

A quadratic function for the parabola is $y = -2(x + 2)^2 + 3$.

EXAMPLE 2 Write a quadratic function in intercept form

Write a quadratic function whose graph has x-intercepts −2 and 1, and point (−1, −4).

Solution

$y = a(x - p)(x - q)$	Intercept form
$y = a(x + 2)(x - 1)$	Substitute −2 for p and 1 for q.
$-4 = a(-1 + 2)(-1 - 1)$	Substitute −1 for x and −4 for y.
$-4 = -2a$	Simplify coefficient of a.
$2 = a$	Solve for a.

A quadratic function for the parabola is $y = 2(x + 2)(x - 1)$.

Exercises for Examples 1 and 2

Write a quadratic function whose graph has the given characteristics.

1. vertex: (1, −4) $y=(x-1)^2-4$
   point on graph: (0, −3)

2. vertex: (−2, 1) $y=-(x+2)^2+1$
   point on graph: (−1, 0)

3. x-intercepts: −3, 1
   point on graph: (−2, −6)
   $y=2(x+3)(x-1)$

4. x-intercepts: −4, 2
   point on graph: (−2, 16)
   $y=-2(x+4)(x-2)$

LESSON 4.10