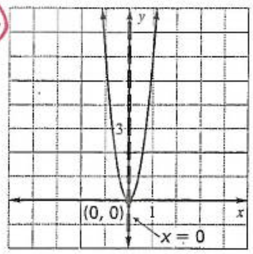
Name Key Date

Study Guide *continued*

For use with pages 236–244

1.


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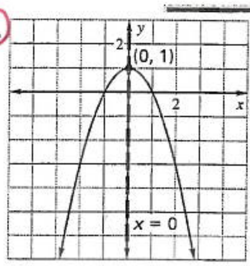

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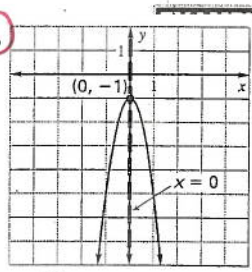

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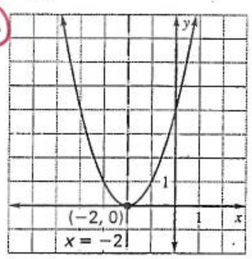

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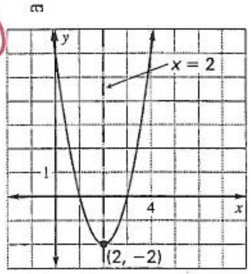

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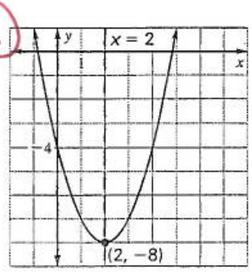


Exercises for Examples 1 and 2

Graph the function. Label the vertex and axis of symmetry.

1. $y = 6x^2$ **2.** $y = -x^2 + 1$ **3.** $y = -4x^2 - 1$

4. $y = x^2 + 4x + 4$ **5.** $y = 2x^2 - 8x + 6$ **6.** $y = x^2 - 4x - 4$

← see left

E3 Find the minimum or maximum value

Tell whether the function $y = -2x^2 + 8x - 7$ has a *minimum value* or a *maximum value*. Find the minimum or maximum value.

Because $a < 0$, the function has a maximum value. The coordinates of the vertex are $x = -\frac{b}{2a} = -\frac{8}{(-4)} = 2$ and $y = -2(2)^2 + 8(2) - 7 = 1$. The maximum value is $y = 1$.

E4 Find the maximum value of a quadratic function

Revenue A street vendor sells about 20 shirts each day when she charges \$8 per shirt. If she decreases the price by \$1, she sells about 10 more shirts each day. How can she maximize daily revenue?

STEP 1 **Define** the variables. Let x represent the price reduction and $R(x)$ represent the daily revenue.

STEP 2 **Write** a verbal model. Then write and simplify a quadratic function.

Revenue (dollars)	=	Price (dollars/shirt)	•	Number of shirts
$R(x)$	=	$(8 - x)$	•	$(20 + 10x)$

$$R(x) = 160 - 20x + 80x - 10x^2$$
$$R(x) = -10x^2 + 60x + 160$$

STEP 3 **Find** the coordinates $(x, R(x))$ of the vertex.

$$x = -\frac{b}{2a} = -\frac{60}{2(-10)} = 3 \qquad \text{Find } x\text{-coordinate.}$$

$$R(3) = -10(3)^2 + 60(3) + 160 = 250 \qquad \text{Evaluate } R(3).$$

The vertex is (3, 250). The vendor should reduce the price by \$3 to maximize daily revenue.

Exercises for Examples 3 and 4

Tell whether the function has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.

7. $y = 2x^2 + 8x - 3$
Minimum
$\frac{-8}{2(2)} = -2$ $2(-2)^2 + 8(-2) - 3$ $\boxed{-11}$

8. $y = -x^2 + 6x - 8$
Maximum $\boxed{1}$
$\frac{-6}{2(-1)}$ $-(3)^2 + 6(3) - 8$

9. $y = -x^2 - 4x - 12$
$-(-2)^2 - 4(-2) - 12$ $\boxed{-8}$
Maximum
$\frac{4}{2(-1)}$

Name Key Date

Study Guide

For use with pages 245–251

GOAL **Graph quadratic equations in vertex form or intercept form.**

> ### Vocabulary
> The **vertex form** of a quadratic equation is given by $y = a(x - h)^2 + k$.
>
> The **intercept form** of a quadratic equation is given by $y = a(x - p)(x - q)$.

EXAMPLE 1 Graph a quadratic function

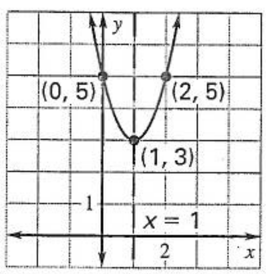
a. Graph $y = 2(x - 1)^2 + 3$, a quadratic function in vertex form.

STEP 1 **Identify** $a = 2$, $h = 1$, and $k = 3$. Because $a > 0$, the parabola opens up.

STEP 2 **Plot** the vertex $(1, 3)$ and draw $x = 1$, the axis of symmetry.

STEP 3 **Evaluate** y for $x = 0$ and $x = 2$. Plot the points $(0, 5)$ and $(2, 5)$.

STEP 4 **Draw** a parabola through the points.



b. Graph $y = 3(x - 4)(x + 2)$, a quadratic function in intercept form.

STEP 1 **Identify** the x-intercepts. Because $p = 4$ and $q = -2$, $(4, 0)$ and $(-2, 0)$ are the x-intercepts.

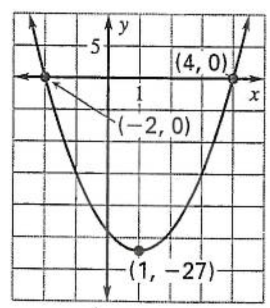
STEP 2 **Find** the coordinates of the vertex.

$$x = \frac{p + q}{2} = \frac{4 + (-2)}{2} = \frac{2}{2} = 1$$

$$y = 3(1 - 4)(1 + 2) = -27$$

The vertex is $(1, -27)$.

STEP 3 **Draw** a parabola through the vertex and the points where the x-intercepts occur.



Exercises for Example 1

Graph the function. Label the vertex, axis of symmetry, and the x-intercepts.

1. $y = (x + 1)^2$

2. $y = -(x - 1)^2$

3. $y = 2(x + 2)^2$

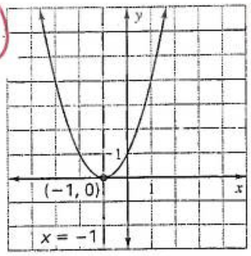
4. $y = (x + 1)(x - 1)$

5. $y = 2(x - 2)(x - 3)$

6. $y = -(x + 4)(x + 2)$

← See left

1



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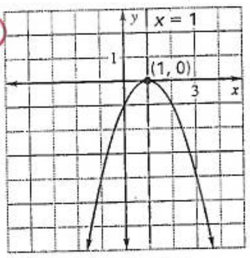

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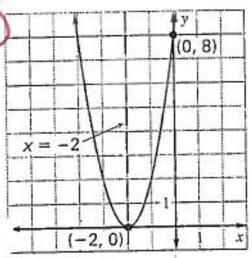

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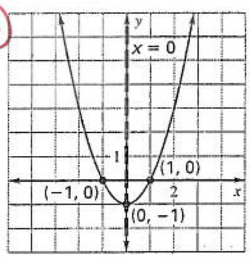

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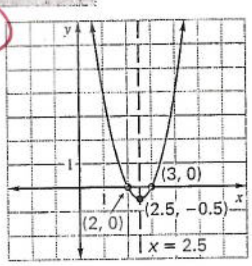

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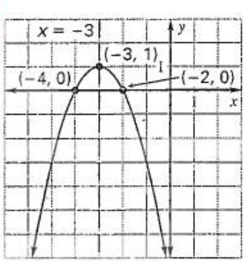

Name Key Date

LESSON 4.2 **Study Guide** *continued*
For use with pages 245–251

(8)
$y=(x+1)(x+1)$

(9)
$y=-1[(x-1)(x-1)]$
$=-1[x^2-2x+1]$

(10)
$y=2[(x+2)(x+2)]$
$=2[x^2+4x+4]$

(12)
$y=2[x^2-5x+6]$

(13)
$y=-1[x^2+5x+6]$

EXAMPLE 2 Use a quadratic model in vertex form

Bridges A bridge is designed with cables that connect two towers that rise above a roadway. Each cable is modeled by

$$y=\frac{(x-1600)^2}{6800}+30.$$

Find the distance between the towers.

Solution

The vertex of the parabola is (1600, 30). The cable's lowest point is 1600 feet from either tower. The distance between the towers is $d=2(1600)=3200$ feet.

Exercise for Example 2

7. Rework Example 2 where $y=\frac{(x-1500)^2}{6500}+28$.

(1500, 28) ; lowest pt 1500 ft from either tower distance betwee is 3000 fee

EXAMPLE 3 Change quadratic functions to standard form

Write the quadratic function in standard form.

a. $y=3(x-1)(x+2)$

b. $y=(x+1)^2+2$

Solution

a. $y=3(x-1)(x+2)$	Write original function in intercept form.
$=3(x^2+2x-x-2)$	Multiply using FOIL.
$=3(x^2+x-2)$	Combine like terms.
$=3x^2+3x-6$	Distributive property
b. $y=(x+1)^2+2$	Write original function in vertex form.
$=(x+1)(x+1)+2$	Rewrite $(x+1)^2$.
$=(x^2+x+x+1)+2$	Multiply using FOIL.
$=x^2+2x+3$	Combine like terms.

Exercises for Example 3

Write the quadratic function in standard form.

8. $y=(x+1)^2$ $y=x^2+2x+1$

9. $y=-(x-1)^2$ $y=-1x^2+2x-1$

10. $y=2(x+2)^2$ $y=2x^2+8x+8$

11. $y=(x+1)(x-1)$ $y=x^2-1$

12. $y=2(x-2)(x-3)$
$y=2x^2-10x+12$

13. $y=-(x+3)(x+2)$
$y=-1x^2-5x-6$

Name Key Date

LESSON 4.3

Study Guide

For use with pages 252–258

GOAL **Solve quadratic equations.**

> ### Vocabulary
> A **monomial** is an expression that is a number, a variable, or the product of a number and one or more variables.
>
> A **binomial** is the sum of two monomials.
>
> A **trinomial** is the sum of three monomials.
>
> A **quadratic equation** in one variable can be written as $ax^2 + bx + c = 0$ where $a \neq 0$.
>
> A solution of a quadratic equation is called the **root of the equation.**
>
> Because a function's value is zero when $x = p$ and $x = q$, the numbers p and q are also called the **zeros of the function.**

EXAMPLE 1 Factor trinomials of the form $x^2 + bx + c$

Factor the expression $w^2 - 2w - 15$.

Solution

You want $w^2 - 2w - 15 = (w + m)(w + n)$ where $mn = -15$ and $m + n = -2$.

Factors of -15: m, n	$-1, 15$	$1, -15$	$-3, 5$	$3, -5$
Sum of factors: $m + n$	14	-14	2	-2

Notice that $m = 3$ and $n = -5$. So, $w^2 - 2w - 15 = (w + 3)(w - 5)$.

EXAMPLE 2 Factor with special patterns

Factor the expression.

a. $g - 20g + 100 = g^2 - 2(g)(10) + 10^2$ Perfect square trinomial

$= (g - 10)^2$

b. $z^2 - 64 = z^2 - 8^2$ Difference of two squares

$= (z + 8)(z - 8)$

Exercises for Examples 1 and 2

Factor the expression. If the expression cannot be factored, say so.

1. $y^2 + 3y - 4$ $(y+4)(y-1)$

2. $j^2 - 11j + 30$ $(j-6)(j-5)$

3. $s^2 + s - 5$ Not factorable prime

4. $s^2 + 4$ Not factorable prime

5. $d^2 + 14d + 49$ $(d+7)(d+7)$ $(d+7)^2$

6. $25a^2 - k^2$ $(5a+k)(5a-k)$

Name Key Date

LESSON 4.3

Study Guide *continued*

For use with pages 252–258

EXAMPLE 3 Find the roots of an equation

Find the roots of $x^2 - 13x + 42 = 0$.

$x^2 - 13x + 42 = 0$	Write original equation.
$(x - 6)(x - 7) = 0$	Factor.
$x - 6 = 0$ or $x - 7 = 0$	Zero product property
$x = 6$ or $x = 7$	Solve for x.

The roots are 6 and 7.

Exercises for Example 3

Find the roots of the equation.

7. $x^2 - 81 = 0$ $(x-9)(x+9)$ $\boxed{9, -9}$

8. $d^2 + d - 2 = 0$ $(d+2)(d-1)$ $\boxed{-2, 1}$

9. $p^2 + 6p + 9 = 0$ $(p+3)(p+3)$ $\boxed{-3}$

EXAMPLE 4 Use a quadratic equation as a model

Gardening A rectangular garden measures 10 feet by 15 feet. By adding x feet to the width and x feet to the length, the area is doubled. Find the new dimensions of the garden.

10

$2(4)(6) = (4+x)(6+x)$

$48 = 24 + 10x + x^2$

$0 = x^2 + 10x - 24$

$0 = (x+12)(x-2)$

$= \cancel{-12}$ 2

increased by 2 feet

6 feet x 8 feet

New area	=	New length	·	New width
$2(10)(15) =$		$(10 + x)$	·	$(15 + x)$

$300 = 150 + 25x + x^2$	Multiply using FOIL.
$0 = x^2 + 25x - 150$	Write in standard form.
$0 = (x + 30)(x - 5)$	Factor.
$x + 30 = 0$ or $x - 5 = 0$	Zero product property
$x = -30$ or $x = 5$	Solve for x.

Reject the negative value. The garden's width and length should each be increased by 5 feet. The new dimensions are 15 feet by 20 feet.

EXAMPLE 5 Find the zeros of quadratic functions

Find the zeros of $y = x^2 + 3x - 28$ by rewriting the function in intercept form.

$y = x^2 + 3x - 28$	Write original function.
$= (x + 7)(x - 4)$	Factor.

The zeros of the function are −7 and 4.

Exercises for Examples 4 and 5

10. Rework Example 4 where a rectangular garden measures 4 feet by 6 feet.

11. Find the zeros of $y = x^2 - 100$ by rewriting the function in intercept form.

$y = (x+10)(x-10)$

$\boxed{x = -10}$ $\boxed{x = 10}$

LESSON 4.3

Name Key Date

LESSON 4.4

Study Guide

For use with pages 259–265

1. $2(x^2-7x+6)$

2. $3(x^2+8x+7)$

3. $6(x^2-7x+12)$

4. $4(x^2+x-6)$

5. $5(x^2-4x-5)$

6. $3(x^2-4x-12)$

GOAL **Use factoring to solve equations of the form $ax^2 + bx + c = 0$.**

EXAMPLE 1 Factor $ax^2 + bx + c$ where $c < 0$

Factor $2x^2 - x - 3$.

You want $2x^2 - x - 3 = (kx + m)(\ell x + n)$ where k and ℓ are factors of 2 and m and n are factors of -3. Because $mn < 0$, m and n have opposite signs.

k, ℓ	2, 1	2, 1	2, 1	2, 1
m, n	3, −1	−1, 3	−3, 1	1, −3
$(kx + m)(\ell x + n)$	$(2x + 3)(x - 1)$	$(2x - 1)(x + 3)$	$(2x - 3)(x + 1)$	$(2x + 1)(x - 3)$
$ax^2 + bx + c$	$2x^2 + x - 3$	$2x^2 + 5x - 3$	$2x^2 - x - 3$	$2x^2 - 5x - 3$

The correct factorization is $(2x - 3)(x + 1)$.

EXAMPLE 2 Factor with special patterns and monomials

Factor the expression.

a. $6t^2 - 24 = 6(t^2 - 4)$ Factor out monomial.

$= 6(t + 2)(t - 2)$ Difference of two squares

b. $3m^2 - 18m + 27 = 3(m^2 - 6m + 9)$ Factor out monomial.

$= 3(m - 3)^2$ Perfect square trinomial

EXAMPLE 3 Solve a quadratic equation

$4s^2 + 11s + 8 = 3s + 4$	Original equation
$4s^2 + 8s + 4 = 0$	Write in standard form.
$s^2 + 2s + 1 = 0$	Divide each side by 4.
$(s + 1)^2 = 0$	Factor.
$s + 1 = 0$	Zero product property
$s = -1$	Solve for s.

Exercises for Examples 1, 2, and 3

Factor the expression. If the expression cannot be factored, say so.

1. $2x^2 - 14x + 12$
$2(x-6)(x-1)$

2. $3x^2 + 24x + 21$
$3(x+7)(x+1)$

3. $6x^2 - 42x + 72$
$6(x-3)(x-4)$

4. $4x^2 + 4x - 24$
$4(x+3)(x-2)$

5. $5x^2 - 20x - 25$
$5(x-5)(x+1)$

6. $3x^2 - 12x - 36$
$3(x-6)(x+2)$

Solve the equation.

7. $25x^2 - 9 = 0$
$25x^2 = 9$
$\sqrt{x^2} = \sqrt{\frac{9}{25}}$
$\boxed{x = \pm\frac{3}{5}}$

8. $3x^2 - 12x + 12 = 0$
$3(x^2-4x+4) = 0$
$3(x-2)(x-2) = 0$
$\boxed{x = 2}$

9. $5x^2 - 15x - 20 = 0$
$5(x^2-3x-4) = 0$
$5(x-4)(x+1) = 0$
$\boxed{x = 4}$

Name Key Date

LESSON 4.4

Study Guide *continued*

For use with pages 259–265

LESSON 4.4

EXAMPLE 4 Use a quadratic equation as a model

Paintings The area of a painting is 24 square inches and the length is 5 inches more than the width. Find the length of the painting.

Solution

Write a verbal model. Let x represent the width and write an equation.

Area of painting (square inches)	=	Width of painting (inches)	$\cdot$	Length of painting (inches)
24	=	x	$\cdot$	$(x + 5)$

$0 = x^2 + 5x - 24$ Write in standard form.

$0 = (x + 8)(x - 3)$ Factor.

$x + 8 = 0$ or $x - 3 = 0$ Zero product property

$x = -8$ or $x = 3$ Solve for x.

Reject the negative value, $x = -8$. The length is $x = 3 + 5$ inches or 8 inches.

EXAMPLE 5 Solve a multi-step problem

Bicycles A bicycle shop sells about 18 bikes per week when it charges \$100 per bike. For each increase of \$10, the shop sells 3 less bikes per week. How much should the shop charge to maximize sales?

STEP 1 **Define** the variables. Let x represent the price increase and $R(x)$ represent the weekly revenue.

STEP 2 **Write** a verbal model. Then write and simplify a quadratic equation.

Weekly sales (dollars)	=	Number of bikes sold (bikes)	$\cdot$	Price of bike (dollars/bike)
$R(x)$	=	$(18 - 3x)$	$\cdot$	$(100 + 10x)$

$R(x) = -30(x - 6)(x + 10)$

STEP 3 **Identify** the zeros and find their average. The zeros are 6 and -10. The average of the zeros is -2. To maximize revenue, the shop should charge $100 + 10(-2) = \$80$.

STEP 4 **Find** the maximum weekly revenue.
$R(-2) = -30(-2 - 6)(-2 + 10) = \$1920$.
The shop should charge \$80 per bike to maximize weekly revenue.
The maximum weekly revenue is \$1920.

10

$24 = x \cdot (x+2)$

$24 = x^2 + 2x$

$0 = x^2 + 2x - 24$

$= (x+12)(x-10)$

$\cancel{-12}$ 10

length $10 + 2$ $\boxed{12 \text{ in}}$

11

$R(x) = (18 - 9x)(100 + 10x)$

$= -9(x-2)\,10(x+10)$

$= -90(x-2)(x+10)$

$= \frac{2 + -10}{2}$

$= -4$

$100 + 10(-4) = 60$

$-90(-4-2)(-4+10) = 3240$

Exercises for Examples 4 and 5

10. Rework Example 4 where the length is 2 inches more than the width.

11. Rework Example 5 where for each increase of \$10, the shop sells 9 less bikes per week.

To maximize revenue, charge \$60 per bike
Maximum revenue is \$3240.

Name Key Date

LESSON 4.5

Study Guide

For use with pages 266–271

GOAL **Solve quadratic equations by finding square roots.**

Vocabulary

A number r is a **square root** of a number s if $r^2 = s$.

The expression $\sqrt{s}$ is called a **radical**. The symbol $\sqrt{\ }$ is a radical sign and the number s beneath the radical sign is the **radicand** of the expression.

To **rationalize a denominator** $\sqrt{b}$ of a fraction, multiply the numerator and denominator by $\sqrt{b}$. To **rationalize a denominator** $a + \sqrt{b}$ of a fraction, multiply the numerator and denominator by $a - \sqrt{b}$, and to **rationalize a denominator** $a - \sqrt{b}$ of a fraction, multiply the numerator and denominator by $a + \sqrt{b}$.

The expressions $a + \sqrt{b}$ and $a - \sqrt{b}$ are called **conjugates.**

EXAMPLE 1 Use properties of square roots

Simplify the expression.

a. $\sqrt{8} \cdot \sqrt{6} = \sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

b. $\sqrt{\frac{5}{36}} = \frac{\sqrt{5}}{\sqrt{36}} = \frac{\sqrt{5}}{6}$

EXAMPLE 2 Rationalize denominators of fractions

Simplify the expression.

a. $\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

b. $\frac{5}{3 - \sqrt{7}} = \frac{5}{3 - \sqrt{7}} \cdot \frac{3 + \sqrt{7}}{3 + \sqrt{7}}$

$= \frac{15 + 5\sqrt{7}}{9 + 3\sqrt{7} - 3\sqrt{7} - 7}$

$= \frac{15 + 5\sqrt{7}}{2}$

Exercises for Examples 1 and 2

Simplify the expression.

1. $\sqrt{200}$ $\boxed{10\sqrt{2}}$

2. $\sqrt{147}$ $\boxed{7\sqrt{3}}$

3. $\sqrt{5} \cdot \sqrt{50}$ $\to \sqrt{250}$ $\boxed{5\sqrt{10}}$

4. $\sqrt{\frac{13}{121}}$ $\boxed{\frac{\sqrt{13}}{11}}$

5. $\sqrt{\frac{7}{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{35}}{5}}$

6. $\frac{2}{\sqrt{3} + 5} \cdot \frac{\sqrt{3} - 5}{\sqrt{3} - 5} = \frac{2\sqrt{3} - 10}{9 - 5}$

$\frac{2\sqrt{3} - 10}{4}$

$\frac{\cancel{2}(\sqrt{3} - 5)}{\cancel{2} \cdot 2}$

$\boxed{\frac{\sqrt{3} - 5}{2}}$