

LESSON
6.1**Study Guide** *continued*
*For use with pages 356–363***Exercises for Examples 1 and 2****Simplify the ratio.**

1. $\frac{60 \text{ mi}}{51 \text{ mi}}$

$\frac{20}{17}$

2. 7 cm : 14 mm
units?

$5:1$

3. A triangle's angle measures are in the extended ratio of 5 : 9 : 16.
-
- Find the measures of the angles.

$30^\circ, 54^\circ, 96^\circ$

EXAMPLE 3 **Solve a proportion****Solve the proportion** $\frac{x}{8} = \frac{5}{4}$.**Solution**

$\frac{x}{8} = \frac{5}{4}$

Write original proportion.

$4 \cdot x = 8 \cdot 5$

Cross Products Property

$4x = 40$

Multiply.

$x = 10$

Divide each side by 4.

EXAMPLE 4 **Find a geometric mean****Find the geometric mean of 45 and 5.****Solution**

$x = \sqrt{ab}$

Definition of geometric mean

$= \sqrt{45 \cdot 5}$

Substitute 45 for a and 5 and b .

$= \sqrt{225}$

Multiply.

$= 15$

Simplify.

The geometric mean of 45 and 5 is 15.

Exercises for Examples 3 and 4**Solve the proportion.**

4. $\frac{a}{12} = \frac{5}{3}$

$a = 20$

5. $\frac{6}{7} = \frac{30}{x}$

$x = 35$

6. $\frac{9}{y} = \frac{3}{7}$

$y = 21$

Find the geometric mean of the two numbers.

7. 3 and 27

9

8. 40 and 5

$10\sqrt{2}$

9. 6 and 15

$3\sqrt{10}$

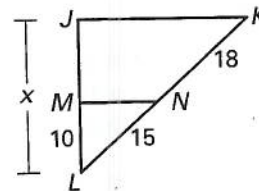
LESSON 6.2

Study Guide *continued*
For use with pages 364–370

Exercises for Example 1

In the diagram, $\frac{JM}{ML} = \frac{KN}{NL}$.

1. Find JL . 22
2. Find JM . 12



EXAMPLE 2 Find the scale of a drawing

Highway A highway on a map is 9 inches long. The actual highway is 36 miles long. What is the scale of the map?

Solution

To find the scale, write the ratio of a length on the map to an actual length, then rewrite the ratio so that the denominator is 1.

$$\frac{\text{length on map}}{\text{length of highway}} = \frac{9 \text{ in.}}{36 \text{ mi}} = \frac{9 \text{ in.} \div 36}{36 \text{ mi} \div 36} = \frac{0.25 \text{ in.}}{1 \text{ mi}}$$

The scale of the map is 0.25 inch : 1 mile.

EXAMPLE 3 Use a scale drawing

Blueprint An architect's blueprint of a floor plan for a new condominium has a scale of 1.5 centimeters : 1 foot. With a ruler you measure the width of the kitchen on the blueprint to be about 22.5 centimeters. What is the actual width of the kitchen?

Solution

Let x represent the actual width in feet.

$$\frac{22.5 \text{ cm}}{x \text{ ft}} = \frac{1.5 \text{ cm}}{1 \text{ ft}} \quad \begin{array}{l} \leftarrow \text{width on blueprint} \\ \leftarrow \text{actual width} \end{array}$$

$$1.5x = 22.5 \quad \text{Cross Products Property}$$

$$x = 15 \quad \text{Solve for } x.$$

The actual width of the kitchen is about 15 feet.

Exercises for Examples 2 and 3

3. A river on a map is 12.5 centimeters long. The actual river is 2.5 miles long. What is the scale of the map? 5 cm : 1 mi
4. In Example 3, the width of the actual living room is 22 feet. What is the width of the living room on the blueprint? 33 cm

$$\frac{1.5 \text{ cm}}{1 \text{ ft}} = \frac{x}{22 \text{ ft}}$$

LESSON 6.3

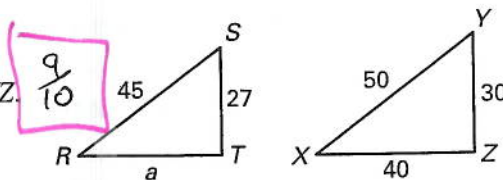
Study Guide *continued*

For use with pages 371–379

Exercises for Examples 1 and 2

In the diagram, $\triangle RST \sim \triangle XYZ$.

- Find the scale factor of $\triangle RST$ to $\triangle XYZ$.
- Find the value of a .



EXAMPLE 3 Find perimeters of similar figures

In the diagram, $ABCD \sim FGHI$.

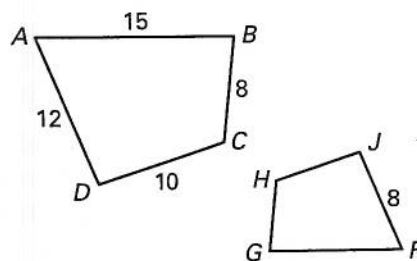
- Find the scale factor of $FGHI$ to $ABCD$.
- Find the perimeter of $FGHI$.

Solution

- Because the figures are similar, the scale factor is the ratio of corresponding sides.

$$\frac{FI}{AD} = \frac{8}{12} = \frac{2}{3}$$

- The perimeter of $ABCD$ is 45. Let x be the perimeter of $FGHI$. Using Theorem 6.1, you can write the proportion $\frac{x}{45} = \frac{2}{3}$. So, the perimeter of $FGHI$ is $x = 30$.



EXAMPLE 4 Use a scale factor

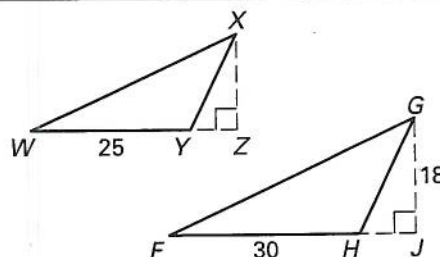
In the diagram, $\triangle WXY \sim \triangle FGH$. Find the length of the altitude \overline{XZ} .

Solution

First, find the scale factor of $\triangle WXY$ to $\triangle FGH$.

$$\frac{WY}{FH} = \frac{25}{30} = \frac{5}{6}$$

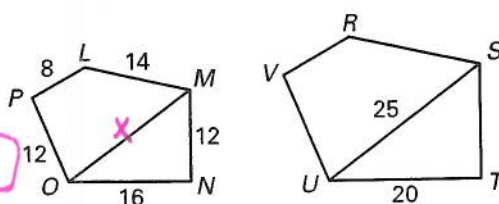
Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the proportion $\frac{XZ}{GJ} = \frac{5}{6}$. Then substitute 18 for GJ and solve for XZ to find that the length of the altitude \overline{XZ} is 15.



Exercises for Examples 3 and 4

In the diagram, $LMNOP \sim RSTUV$.

- Find the scale factor of $RSTUV$ to $LMNOP$.
- Find the perimeter of $RSTUV$.
- Find the length of diagonal \overline{MO} .



Handwritten notes for the exercises: $\frac{5}{4}$ (scale factor), 77.5 (perimeter), and $\frac{25}{x} = \frac{20}{16}$ (proportion for diagonal length).

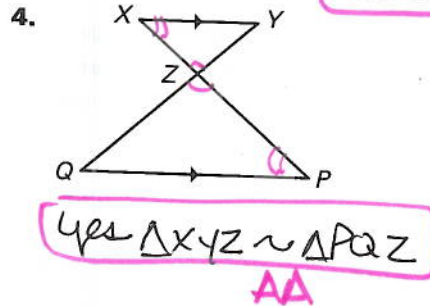
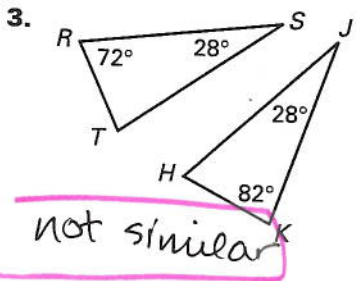
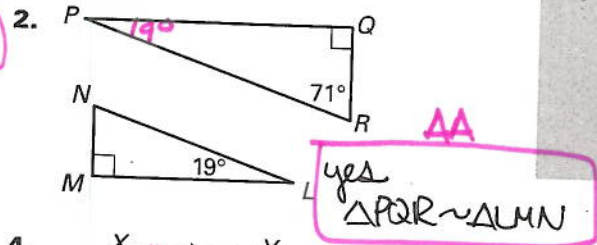
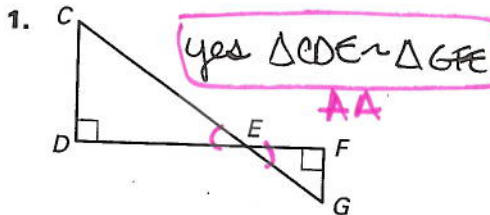
LESSON 6.4

Study Guide *continued*
For use with pages 381-387

LESSON 6.4

Exercises for Examples 1 and 2

Determine whether the triangles are similar. If they are, write a similarity statement.

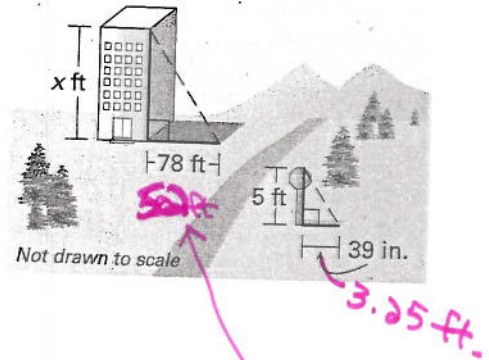


EXAMPLE 3 Use indirect measurement

Shadows A building casts a shadow that is 78 feet long. At the same time, a nearby sign post that is 5 feet tall casts a shadow that is 39 inches long. How tall is the building?

Solution

The building and the sign post form sides of two right angles with the ground, as shown in the diagram. The sun's rays hit the building and the sign post at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate. Write 39 inches as 3.25 feet so that all the dimensions are in feet. Then you can write the proportion $\frac{x \text{ ft}}{5 \text{ ft}} = \frac{78 \text{ ft}}{3.25 \text{ ft}}$ to find that $x = 120$. So, the building is 120 feet tall.



Exercises for Example 3

- In Example 3, suppose a different building casts a shadow that is 52 feet long. How tall is the building? **80 ft.** $\frac{x}{52} = \frac{5}{3.25}$
- You and your friend are standing next to one another outside. Your shadow is 23 inches long and your friend's shadow is 24 inches long. You are 5 feet 5 inches tall. How tall is your friend? **68 in. or 5 ft. 8 in.**

LESSON
6.5

Study Guide

For use with pages 388–395

GOAL Use the SSS and SAS Similarity Theorems.

Vocabulary

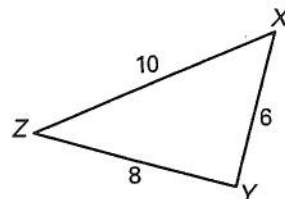
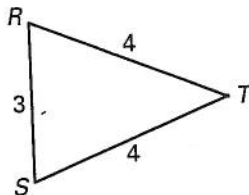
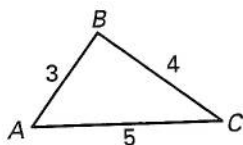
Theorem 6.2 Side-Side-Side (SSS) Similarity Theorem: If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

Theorem 6.3 Side-Angle-Side (SAS) Similarity Theorem: If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

LESSON 6.5

EXAMPLE 1 Use the SSS Similarity Theorem

Is either $\triangle RST$ or $\triangle XYZ$ similar to $\triangle ABC$?



Solution

Compare $\triangle ABC$ and $\triangle RST$ by finding ratios of corresponding side lengths.

Shortest sides

$$\frac{AB}{RS} = \frac{3}{3} = 1$$

Longest sides

$$\frac{CA}{RT} = \frac{5}{4}$$

Remaining sides

$$\frac{BC}{ST} = \frac{4}{4} = 1$$

The ratios are not all equal, so $\triangle ABC$ and $\triangle RST$ are not similar.

Compare $\triangle ABC$ and $\triangle XYZ$ by finding ratios of corresponding side lengths.

Shortest sides

$$\frac{AB}{XY} = \frac{3}{6} = \frac{1}{2}$$

Longest sides

$$\frac{CA}{ZX} = \frac{5}{10} = \frac{1}{2}$$

Remaining sides

$$\frac{BC}{YZ} = \frac{4}{8} = \frac{1}{2}$$

All of the ratios are equal, so $\triangle ABC \sim \triangle XYZ$.

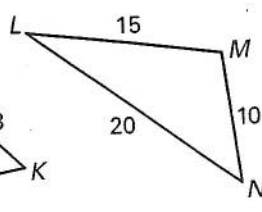
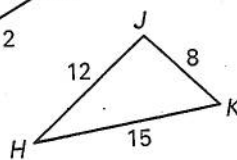
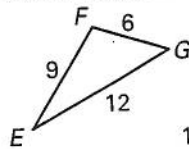
Exercise for Example 1

- Which of the three triangles are similar? Write a similarity statement.

$$\triangle EFG \sim \triangle LMN$$

$$\frac{10}{6} = \frac{15}{9} = \frac{20}{12}$$

$$\frac{5}{3} = \frac{5}{3} = \frac{5}{3}$$

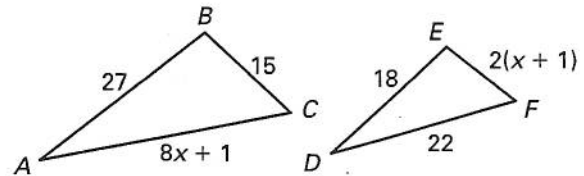


LESSON 6.5

Study Guide *continued*
For use with pages 388–395

EXAMPLE 2 Use the SSS Similarity Theorem

Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



Solution

STEP 1 Find the value of x that makes corresponding side lengths proportional.

$$\frac{27}{18} = \frac{15}{2(x+1)}$$

Write proportion.

$$27 \cdot 2(x+1) = 18 \cdot 15$$

Cross Products Property

$$54x + 54 = 270$$

Simplify.

$$x = 4$$

Solve for x .

STEP 2 Check that the side lengths are proportional when $x = 4$.

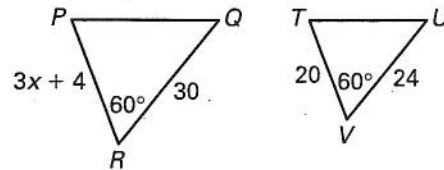
$$AC = 8x + 1 = 33 \quad EF = 2(x + 1) = 10$$

$$\frac{AB}{DE} = \frac{27}{18} = \frac{3}{2} \quad \frac{BC}{EF} = \frac{15}{10} = \frac{3}{2} \quad \frac{AC}{DF} = \frac{33}{22} = \frac{3}{2}$$

When $x = 4$, the triangles are similar by the SSS Similarity Theorem.

EXAMPLE 3 Use the SAS Similarity Theorem

Find the value of x that makes $\triangle PQR \sim \triangle TUV$.



Solution

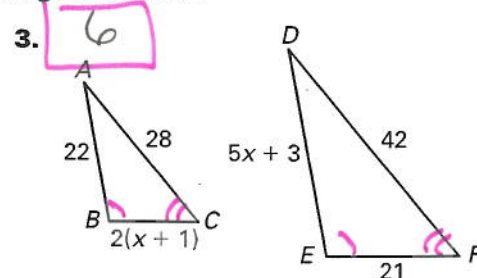
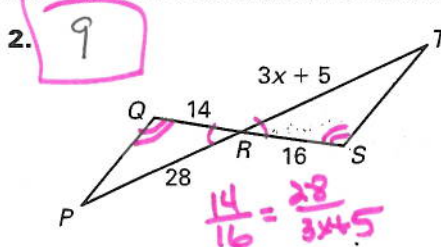
Both $m\angle R$ and $m\angle V$ equal 60° , so $\angle R \cong \angle V$. Next, find the value of x that makes the sides including these angles

proportional. Solving the proportion $\frac{3x+4}{20} = \frac{30}{24}$, you obtain $x = 7$. So, by the SAS

Similarity Theorem, the triangles are similar when $x = 7$.

Exercises for Examples 2 and 3

Find the value of x that makes the triangles similar.



LESSON 6.6

Study Guide *continued*
For use with pages 396–403

EXAMPLE 3 Use Theorem 6.6

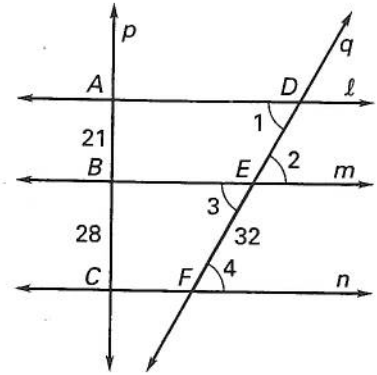
In the diagram, $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are all congruent and $AB = 21$, $BC = 28$, and $EF = 32$. Find the length of DE .

Alternate interior angles are congruent, so $l \parallel m \parallel n$. Use Theorem 6.6.

$$\frac{AB}{BC} = \frac{DE}{EF} \quad \text{Parallel lines divide transversals proportionally.}$$

$$\frac{21}{28} = \frac{DE}{32} \quad \text{Substitute.}$$

$$DE = 24 \quad \text{Solve for } DE.$$



EXAMPLE 4 Use Theorem 6.7

In the diagram, $\angle ABD \cong \angle CBD$. Use the given side lengths to find the length of \overline{AD} .

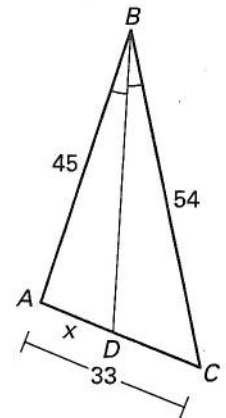
Because \overline{BD} is an angle bisector of $\angle ABC$, you can apply Theorem 6.7. Let $AD = x$. Then $DC = 33 - x$.

$$\frac{DC}{AD} = \frac{BC}{BA} \quad \text{Angle bisector divides opposite side proportionally.}$$

$$\frac{33 - x}{x} = \frac{54}{45} \quad \text{Substitute.}$$

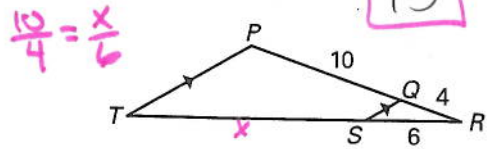
$$54x = 1485 - 45x \quad \text{Cross Products Property}$$

$$x = 15 \quad \text{Solve for } x.$$

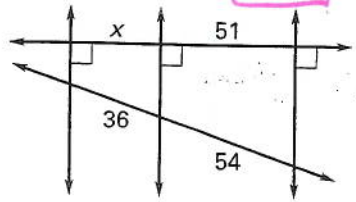


Exercises for Examples 1, 2, 3, and 4

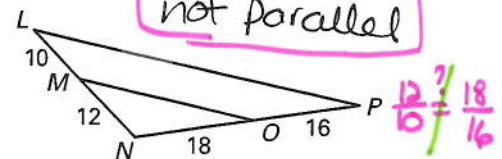
1. Find the length of \overline{ST} . 15



3. Find the value of x . 34



2. Determine whether $\overline{MO} \parallel \overline{LP}$. not parallel



4. Find the value of x . 21

