Name KEY Date ____

LESSON 7.1

Study Guide

For use with pages 426–434

GOAL **Graph and solve systems of linear equations.**

Vocabulary

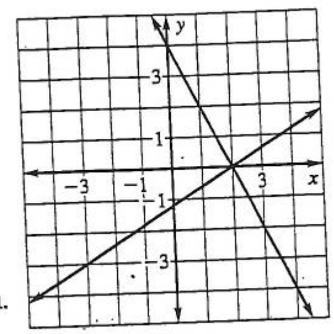
A **system of linear equations,** or simply a *linear system*, consists of two or more linear equations in the same variables.

A **solution of a system of linear equations** in two variables is an ordered pair that satisfies each equation in the system.

LESSON 7.1

EXAMPLE 1 Check the intersection point

Use the graph to solve the system. Then check your solution algebraically.



$2x + y = 4$ Equation 1

$3x - 5y = 6$ Equation 2

Solution

The lines appear to intersect at the point (2, 0).

CHECK Substitute 2 for x and 0 for y in each equation.

Equation 1	Equation 2
$2x + y = 4$	$3x - 5y = 6$
$2(2) + 0 \stackrel{?}{=} 4$	$3(2) - 5(0) \stackrel{?}{=} 6$
$4 + 0 \stackrel{?}{=} 4$	$6 - 0 \stackrel{?}{=} 6$
$4 = 4$ ✓	$6 = 6$ ✓

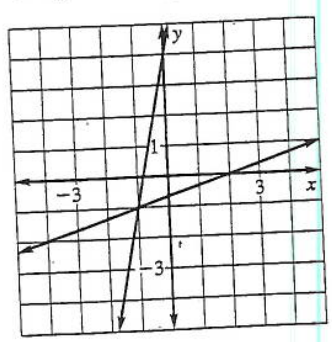
Because the ordered pair (2, 0) is a solution of each equation, it is a solution of the system.

EXAMPLE 2 Use the graph-and-check method

Solve the linear system: $x - 3y = 2$ Equation 1

$-5x + y = 4$ Equation 2

STEP 1 **Graph** both equations.



STEP 2 **Estimate** the point of the intersection. The two lines appear to intersect at $(-1, -1)$.

Name ______________________ Date ______________

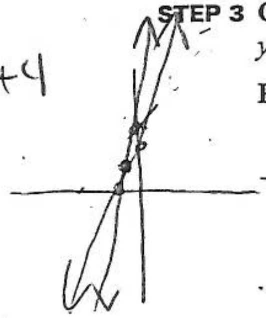
LESSON 7.1

Study Guide *continued*

For use with pages 426–434

① $y=3x+4$

x	y
$-\frac{7}{5}$	0
0	$\frac{7}{2}$



STEP 3 **Check** whether $(-1, -1)$ is a solution by substituting -1 for x and -1 for y in each of the original equations.

Equation 1	**Equation 2**
$x - 3y = 2$	$-5x + y = 4$
$-1 - 3(-1) \stackrel{?}{=} 2$	$-5(-1) + (-1) \stackrel{?}{=} 4$
$-1 + 3 \stackrel{?}{=} 2$	$5 - 1 \stackrel{?}{=} 4$
$2 = 2$ ✓	$4 = 4$ ✓

Because the ordered pair $(-1, -1)$ is a solution of each equation, it is a solution of the system.

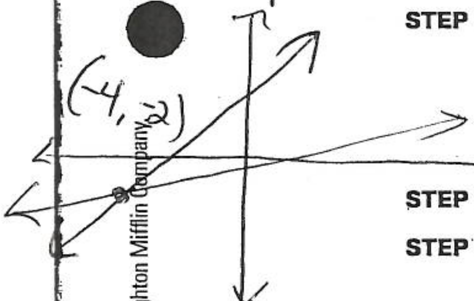
EXAMPLE 3 Solve a multi-step problem

③

x	y
2	0
0	$-\frac{2}{3}$

x	y
$-\frac{20}{7}$	0
0	5

$(-4,-2)$



Delivery Service The Rosebud Flower Shop has a basic delivery charge of \$5 plus a rate of \$.25 per mile. The Beautiful Bouquets Shop has a basic delivery charge of \$7 plus a rate of \$.20 per mile. Determine the number of miles a delivery must be for the charges to be equal.

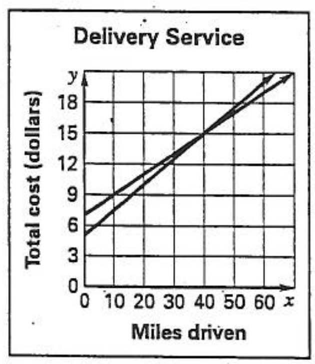
Solution

STEP 1 **Write** a linear system. Let x be the number of miles driven and y be the total cost of the delivery.

$y = 5 + 0.25x$ Equation for Rosebud Flower Shop

$y = 7 + 0.20x$ Equation for Beautiful Bouquets Shop

STEP 2 **Graph** both equations.



STEP 3 **Estimate** the point of intersection. The two lines appear to intersect at (40, 15).

STEP 4 **Check** whether (40, 15) is a solution.

Equation 1	**Equation 2**
$y = 5 + 0.25x$	$y = 7 + 0.20x$
$15 \stackrel{?}{=} 5 + 0.25(40)$	$15 \stackrel{?}{=} 7 + 0.20(40)$
$15 = 15$ ✓	$15 = 15$ ✓

Exercises for Examples 1, 2, and 3

Solve the linear system by graphing. remember graphing is the least accurate method!

1. $-3x + y = 4$ $(-1,1)$
$5x - 2y = -7$

2. $x + \frac{1}{2}y = 4$
$5x + 2y = 18$

3. $2x - 6y = 4$ $(-4,-2)$
$7x - 4y = -20$

4. In Example 3, suppose Rosebud Flower Shop increases its basic charge to \$10, and Beautiful Bouquets raises its basic charge to \$13. Determine when the costs will be equal.

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LESSON 7.2

Study Guide

For use with pages 435–441

GOAL **Solve systems of linear equations by substitution.**

EXAMPLE 1 Use the substitution method

Solve the linear system: $2x + y = 1$ Equation 1

$x + 2y = 5$ Equation 2

Solution

STEP 1 Solve Equation 1 for y.

$2x + y = 1$	Write original Equation 1.
$y = -2x + 1$	Subtract $2x$ from each side.

STEP 2 Substitute $-2x + 1$ for y in Equation 2 and solve for x.

$x + 2y = 5$	Write Equation 2.
$x + 2(-2x + 1) = 5$	Substitute $-2x + 1$ for y.
$x - 4x + 2 = 5$	Distributive property
$-3x + 2 = 5$	Simplify.
$-3x = 3$	Subtract 2 from each side.
$x = -1$	Divide each side by -3.

STEP 3 Substitute -1 for x in the original Equation 1 to find the value of y.

$2x + y = 1$	Write original Equation 1.
$2(-1) + y = 1$	Substitute -1 for x.
$-2 + y = 1$	Simplify.
$y = 3$	Solve for y.

The solution is $(-1, 3)$.

CHECK Substitute -1 for x and 3 for y in each of the original equations.

Equation 1	Equation 2
$2x + y = 1$	$x + 2y = 5$
$2(-1) + 3 \stackrel{?}{=} 1$	$-1 + 2(3) \stackrel{?}{=} 5$
$1 = 1$ ✓	$5 = 5$ ✓

LESSON 7.2

Name ______________________ Date ______________

LESSON 7.2

Study Guide *continued*

For use with pages 435–441

EXAMPLE 2 Use the substitution method

Solve the linear system: $2x + 5y = 5$ Equation 1

$x - 4y = 9$ Equation 2

Solution

STEP 1 **Solve** Equation 2 for x.

$x - 4y = 9$	Write original Equation 2.
$x = 4y + 9$	Revised Equation 2

STEP 2 **Substitute** $4y + 9$ for x in Equation 1 and solve for y.

$2x + 5y = 5$	Write Equation 1.
$2(4y + 9) + 5y = 5$	Substitute $4y + 9$ for x.
$8y + 18 + 5y = 5$	Distributive property
$13y + 18 = 5$	Simplify.
$13y = -13$	Subtract 18 from each side.
$y = -1$	Divide each side by 13.

STEP 3 **Substitute** -1 for y in the revised Equation 2 to find the value of x.

$x = 4y + 9$	Revised Equation 2
$x = 4(-1) + 9$	Substitute -1 for y.
$x = 5$	Simplify.

The solution is $(5, -1)$.

CHECK Substitute 5 for x and -1 for y in each equation.

Equation 1	Equation 2
$2x + 5y = 5$	$x - 4y = 9$
$2(5) + 5(-1) \stackrel{?}{=} 5$	$5 - 4(-1) \stackrel{?}{=} 9$
$5 = 5$ ✓	$9 = 9$ ✓

Exercises for Examples 1 and 2

Solve the linear system using the substitution method.

1. $x + 3y = -10$
$7x - 5y = 34$ $(2, -4)$

2. $8x + 5y = 6$
$5x - y = -21$

3. $6x - 7y = 22$
$x - 4y = -2$ $(6, 2)$

4. $6x + y = 26$
$5x - 2y = -1$

5. $x + 3y = 11$
$5x + 6y = 1$

6. $\frac{3}{2}x + y = 8$
$4x - \frac{1}{2}y = 15$ $(4, 2)$

1
$x = -3y - 10$
$7x - 5y = 34$
$7(-3y - 10) - 5y = 34$
$-21y - 70 - 5y = 34$
$-26y - 70 = 34$
$-26y = 104$
$y = -4$ $x = 2$
$x = -3(-4) - 10$

3
$6x - 7y = 22$
$x = 4y - 2$
$6(4y - 2) - 7y = 22$
$24y - 12 - 7y = 22$
$17y - 12 = 22$
$17y = 34$
$y = 2$
$x = 4(2) - 2$
$x = 6$

6
$y = -\frac{3}{2}x + 8$
$4x - \frac{1}{2}y = 15$
$4x - \frac{1}{2}(-\frac{3}{2}x + 8) = 15$
$4x + \frac{3}{4}x - 4 = 15$
$\frac{16}{4}x + \frac{3}{4}x = 19$
$\frac{4}{19} \cdot \frac{19}{4}x = 19 \cdot \frac{4}{19}$
$x = 4$
$y = -\frac{3}{2} \cdot 4 + 8$
$y = -6 + 8$ $y = 2$

Name ______________________ Date ______________

LESSON 7.3

Study Guide

For use with pages 443–450

GOAL **Solve linear systems by elimination.**

EXAMPLE 1 Use addition to eliminate a variable

Solve the linear system: $2x + 4y = 2$ Equation 1

$4x - 4y = 16$ Equation 2

Solution

STEP 1 **Add** the equations to eliminate one variable.

$$\begin{array}{r} 2x + 4y = 2 \\ 4x - 4y = 16 \\ \hline 6x \quad\;\; = 18 \end{array}$$

STEP 2 Solve for x.

$$x = 3$$

STEP 3 **Substitute** 3 for x in either equation and solve for y.

$2x + 4y = 2$	Write Equation 1.
$2(3) + 4y = 2$	Substitute 3 for x.
$y = -1$	Solve for y.

The solution is $(3, -1)$.

CHECK Substitute 3 for x and -1 for y in each equation.

Equation 1

$2x + 4y = 2$

$2(3) + 4(-1) \stackrel{?}{=} 2$

$2 = 2$ ✓

Equation 2

$4x - 4y = 16$

$4(3) - 4(-1) \stackrel{?}{=} 16$

$16 = 16$ ✓

EXAMPLE 2 Use subtraction to eliminate a variable

Solve the linear system: $7x + 5y = 18$ Equation 1

$7x - 3y = 34$ Equation 2

Solution

STEP 1 **Subtract** the equations to eliminate one variable.

$$\begin{array}{r} 7x + 5y = 18 \\ 7x - 3y = 34 \\ \hline 8y = -16 \end{array}$$

STEP 2 Solve for y.

$$y = -2$$

STEP 3 **Substitute** -2 for y in either equation and solve for x.

$7x + 5y = 18$	Write Equation 1.
$7x + 5(-2) = 18$	Substitute -2 for y.
$x = 4$	Solve for x.

The solution is $(4, -2)$.

Name ______________________ Date ______________

LESSON 7.3

Study Guide *continued*

For use with pages 443–450

EXAMPLE 3 Arrange like terms

Solve the linear system: $6x - 4y = 10$ Equation 1

$13y = 6x + 8$ Equation 2

Solution

STEP 1 **Rewrite** Equation 1 so that the like terms are arranged in columns.

$$\begin{aligned} 6x - 4y &= 10 \\ 13y &= 6x + 8 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 6x - 4y &= 10 \\ -6x + 13y &= 8 \\ \hline 9y &= 18 \\ y &= 2 \end{aligned}$$

STEP 2 **Add** the equations.

STEP 3 **Solve** for y.

STEP 4 **Substitute** 2 for y in either equation and solve for x.

$6x + 4y = 10$	Write Equation 1.
$6x - 4(2) = 10$	Substitute 2 for y.
$x = 3$	Solve for x.

The solution is (3, 2).

Exercises for Examples 1, 2, and 3

Simplify the linear system.

1. $5x + 8y = 36$
   $7x - 8y = 12$ (4, 2)

2. $4x + 5y = 8$
   $-4x - 3y = 0$

3. $9x - 8y = 7$
   $9x + 2y = -13$ (-1, -2)

4. $-4x + 7y = 11$
   $2x + 7y = 47$

5. $9x + 8y = -30$
   $9x = 4y + 42$ (2, -6)

6. $5y = 4x + 3$
   $7x = 36 - 5y$

① $5x + 8y = 36$
$7x - 8y = 12$
$12x = 48$
$\frac{12x}{12} = \frac{48}{12}$
$x = 4$
$5(4) + 8y = 36$
$8y = 16$
$y = 2$

③ $9x - 8y = 7$
$9x + 2y = -13$
$-9x + 8y = -7$
$10y = -20$
$\frac{10y}{10} = \frac{-20}{10}$
$y = -2$
$9x - 8(-2) = 7$
$9x + 16 = 7$
$-16 \quad -16$
$9x = -9$

⑤ $9x + 8y = -30$
$9x - 4y = 42$
$-9x - 8y = 30$
$-12y = 72$
$y = -6$
$9x + 8(-6) = -30$
$9x + -48 = -30$
$9x = 18$
$x = 2$

LESSON 7.3

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LESSON 7.4

Study Guide

For use with pages 451–457

GOAL **Solve linear systems by multiplying first.**

EXAMPLE 1 Multiply one equation, then add

Solve the linear system: $3x - 2y = -4$ Equation 1

$7x - 4y = -6$ Equation 2

Solution

STEP 1 Multiply Equation 1 by -2 so that the coefficients of y are opposites.

$$\begin{aligned} 3x - 2y &= -4 \\ 7x - 4y &= -6 \end{aligned} \xrightarrow{\times(-2)} \begin{aligned} -6x + 4y &= 8 \\ 7x - 4y &= -6 \\ \hline x \quad\quad &= 2 \end{aligned}$$

STEP 2 **Add** the equations.

STEP 3 **Substitute** 2 for x in either equation and solve for y.

$3x - 2y = -4$ Write Equation 1.

$3(2) - 2y = -4$ Substitute 2 for x.

$y = 5$ Solve for y.

The solution is (2, 5).

CHECK Substitute 2 for x and 5 for y in each equation.

Equation 1	**Equation 2**
$3x - 2y = -4$	$7x - 4y = -6$
$3(2) - 2(5) \stackrel{?}{=} -4$	$7(2) - 4(5) \stackrel{?}{=} -6$
$-4 = -4$ ✓	$-6 = -6$ ✓

Exercises for Example 1

Solve the linear system using elimination.

1. $15x + 4y = 25$
   -3($5x - 3y = 30$) ($\frac{1}{3}$, 5)
2. $5x + 3y = 18$
   $9y = -7x + 6$ (6, -4)
3. $4x = 7y + 14$
   $14y = 3x + 7$ (7, 2)

① $15x+4y=25$
$-15x+9y=-90$
$13y=65$
$y=5$
$15x+4(5)=25$
$\frac{15x}{15}=\frac{5}{15}$
$x=\frac{1}{3}$

② $-3(\cancel{5x+3y=18})$
$7x+9y=6$
$-15x+-9y=-54$
$-8x=-48$
$x=6$
$5(6)+3y=18$
$3y=-12$
$y=-4$

③ $2(\cancel{4x-7y=14})$
$-3x+14y=7$
$8x-14y=28$
$5x=35$
$x=7$
$-3(7)+14y=7$
$-21+14y=7$
$14y=28$
$y=2$

Name ______________________ Date ____________

LESSON 7.4

Study Guide *continued*

For use with pages 451–457

EXAMPLE 2 Multiply both equations, then add

Solve the linear system: $5x + 2y = -18$ Equation 1

$7y = 3x + 19$ Equation 2

Solution

STEP 1 **Arrange** the equations so that like terms are in columns.

$5x + 2y = -18$ Write Equation 1.

$-3x + 7y = 19$ Rewrite Equation 2.

STEP 2 **Multiply** Equation 1 by 3 and Equation 2 by 5 so that the coefficients in each equation are the least common multipe of 5 and 3, or 15.

$$5x + 2y = -18 \xrightarrow{\times 3} 15x + 6y = -54$$

$$-3x + 7y = 19 \xrightarrow{\times 5} -15x + 35y = 95$$

STEP 3 **Add** the equations. $41y = 41$

STEP 4 **Solve** for y. $y = 1$

STEP 5 **Substitute** 1 for y in either of the original equations and solve for x.

$5x + 2y = -18$ Write Equation 1.

$5x + 2(1) = -18$ Substitute 1 for y.

$x = -4$ Solve for x.

The solution is $(-4, 1)$.

CHECK Substitute -4 for x and 1 for y in each equation.

Equation 1	**Equation 2**
$5x + 2y = -18$	$7y = 3x + 19$
$5(-4) + 2(1) \stackrel{?}{=} -18$	$7(1) \stackrel{?}{=} 3(-4) + 19$
$-18 = -18$ ✓	$7 = 7$ ✓

Exercises for Example 2

Solve the linear system using elimination.

(2,3) **4.** $9x + 5y = 33$
$12x - 7y = 3$

(4) $-4(9x+5y=33) \rightarrow -36x + -20y = -132$
$3(12x-7y=3) \rightarrow 36x - 21y = 9$
$-41y = -123$
$y = 3$
$9x + 5(3) = 33$
$9x = 18$
$x = 2$

~~**5.** $3x + 7y = 20$~~
~~$5x = -4y + 41$~~

(5,6) **6.** $9y = 10x + 4$
$12x = 5y + 30$

(6) $5(-10x+9y=4) \rightarrow -50x + 45y = 20$
$9(12x-5y=30) \rightarrow 108x - 45y = 270$
$58x = 290$
$x = 5$
$-10(5) + 9y = 4$
$9y = 54$

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LESSON 7.5

Study Guide

For use with pages 459–465

GOAL **Identify the number of solutions of a linear system.**

> **Vocabulary**
>
> A linear system with no solution is called an **inconsistent system.**
>
> A linear system with infinitely many solutions is called a **dependent system.**

EXAMPLE 1 A linear system with no solution

Show that the linear system has no solution.

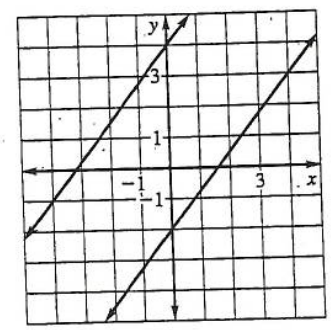
$-5x + 4y = 16$ Equation 1

$5x - 4y = 8$ Equation 2

Solution

Method 1 Graphing

Graph the linear system.



The lines are parallel because they have the same slope but different y-intercepts. Parallel lines do not intersect, so the system has no solution.

Method 2 Elimination

Add the equations.

$$\begin{array}{r} -5x + 4y = 16 \\ 5x - 4y = 8 \\ \hline 0 = 24 \end{array}$$

$\leftarrow$ This is a false statement.

The variables are eliminated and you are left with a false statement regardless of the values of x and y. This tells you that the system has no solution.

EXAMPLE 2 A linear system with infinitely many solutions

Show that the linear system has infinitely many solutions.

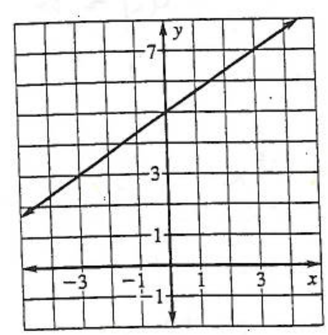
$y = \frac{2}{3}x + 5$ Equation 1

$-2x + 3y = 15$ Equation 2

Solution

Method 1 Graphing

Graph the linear system.



The equations represent the same line, so any point on the line is a solution. So, the linear system has infinitely many solutions.

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Study Guide *continued*

For use with pages 459–465

(1.) $-15x+3(5x+2)=6$
$-15x+15x+6=6$
$6=6$ ✓

Method 2 Substitution

Substitute $\frac{2}{3}x + 5$ for y in Equation 2 and solve for x.

$-2x + 3y = 15$	Write Equation 2.
$-2x + 3\left(\frac{2}{3}x + 5\right) = 15$	Substitute $\frac{2}{3}x + 5$ for y.
$-2x + 2x + 15 = 15$	Distributive property
$15 = 15$	Simplify.

The variables are eliminated and you are left with a statement that is true regardless of the values of x and y. This tells you the system has infinitely many solutions.

Exercises for Examples 1 and 2

Tell whether the linear system has *no solution* or *infinitely many solutions.*

1. $-15x + 3y = 6$
$y = 5x + 2$
infinitely many

2. $-4x + y = 5 \rightarrow y = 4x + 5$
$y = 4x + 3$ $y = 4x + 3$
same slopes
parallel
no solution

EXAMPLE 3

Identify the number of solutions

Without solving the linear system, tell whether the linear system has *one solution, no solution,* or *infinitely many solutions.*

a. $7x - 2y = 9$ Equation 1
$7x - 2y = -1$ Equation 2

b. $3x + y = -10$ Equation 1
$-6x - 2y = 20$ Equation 2

Solution

a. $y = \frac{7}{2}x - \frac{9}{2}$ Write Equation 1 in slope-intercept form.

$y = \frac{7}{2}x + \frac{1}{2}$ Write Equation 2 in slope-intercept form.

Because the lines have the same slope but different y-intercepts, the system has no solution.

b. $y = -3x - 10$ Write Equation 1 in slope-intercept form.

$y = -3x - 10$ Write Equation 2 in slope-intercept form.

The lines have the same slope and y-intercept, so the system has infinitely many solutions.

Exercises for Example 3

find slope

Without solving the linear system, tell whether the linear system has *one solution, no solution,* or *infinitely many solutions.*

3. $x - 3y = 7$
$4x = 12y + 28$
infinitely many

4. $2x + 3y = 17$
$3x + 2y = 14$
one solution

5. $-4x + y = 5$
$-8x - 14y = -28$
one solution

(3) $\cancel{x-3y=7}$
$4x-12y=28$
$-4x+12y=-28$
$0=0$

(4) $\frac{3y}{3}=\frac{-2x}{3}+\frac{17}{3}$
$y=-\frac{2}{3}x+\frac{17}{3}$
$\frac{2y}{2}=\frac{-3x}{2}+\frac{11}{2}$
$y=-\frac{3}{2}x+7$
different slopes

(5) $y=4x+5$
$\frac{-14y}{-14}=\frac{8x}{-14}-\frac{28}{-14}$
$y=-\frac{2}{7}x+2$
different slopes

Name ______________________ Date ______________

LESSON 7.6

Study Guide

For use with pages 466–472

GOAL **Solve systems of linear inequalities in two variables.**

Vocabulary

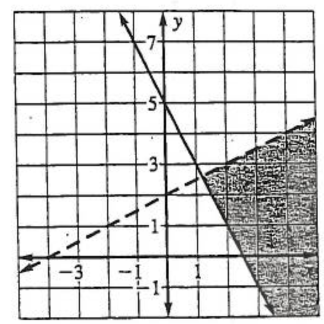
A **system of linear inequalities** in two variables, or simply a *system of inequalities*, consists of two or more linear inequalities in the same variables.

A **solution of a system of linear inequalities** is an ordered pair that is a solution of each inequality in the system.

The **graph of a system of linear inequalities** is the graph of all solutions of the system.

EXAMPLE 1 Graph a system of two linear inequalities

Graph the system of inequalities.



$y < \frac{1}{2}x + 2$ Inequality 1

$y \geq -2x + 5$ Inequality 2

Solution

Graph both inequalities in the same coordinate plane. The graph of the system is the intersection of the two half-planes, which is shown as the shaded region.

CHECK Choose a point in the shaded region, such as (2, 2). To check this solution, substitute 2 for x and 2 for y into each inequality.

Inequality 1	Inequality 2
$y < \frac{1}{2}x + 2$	$y \geq -2x + 5$
$2 \stackrel{?}{<} \frac{1}{2}(2) + 2$	$2 \stackrel{?}{\geq} -2(2) + 5$
$2 < 3$ ✓	$2 \geq 1$ ✓

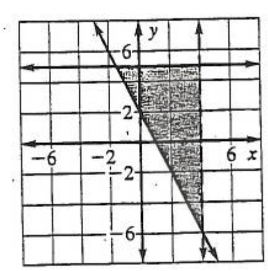
EXAMPLE 2 Graph a system of three linear inequalities

Graph the system of inequalities.

$y \leq 5$ Inequality 1

$x < 4$ Inequality 2

$y \geq -2x + 2$ Inequality 3



Solution

Graph all three inequalities in the same coordinate plane. The graph of the system is the triangular region shown.

LESSON 7.6

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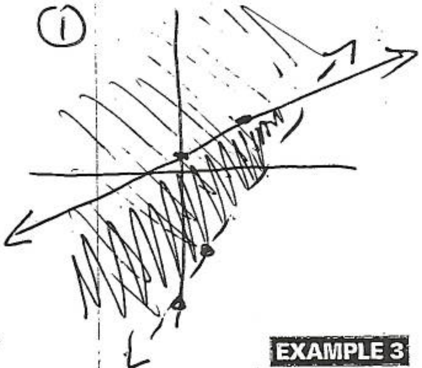
LESSON 7.6

Study Guide *continued*

For use with pages 466–472

(1)



Exercises for Examples 1 and 2

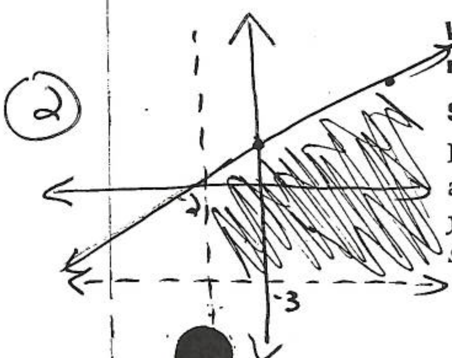
Graph the system of linear inequalities.

1. $y > 3x - 7$
   $y \le \frac{2}{3}x + 1$

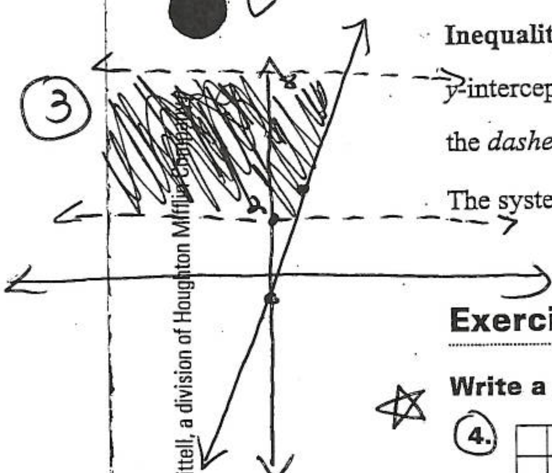
2. $x > -2$
   $y > -3$
   $y \le \frac{3}{5}x + 2$

3. $y > 2$
   $y < 8$
   $y \ge 4x - 1$

(2)



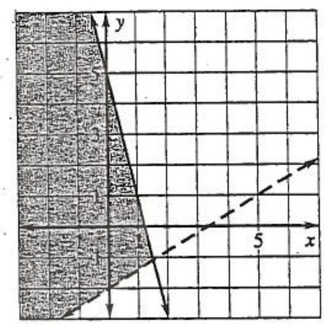
(3)



EXAMPLE 3

Write a system of linear inequalities

Write a system of inequalities for the shaded region.



Solution

Inequality 1 One boundary for the shaded region has a slope of -4 and a y-intercept of 5. So, its equation is $y = -4x + 5$. Because the shaded region is *below* the *solid line*, the inequality is $y \le -4x + 5$.

Inequality 2 Another boundary line for the shaded region has a slope of $\frac{3}{5}$ and a y-intercept of -2. So, its equation is $y = \frac{3}{5}x - 2$. Because the shaded region is *above* the *dashed line*, the inequality is $y > \frac{3}{5}x - 2$.

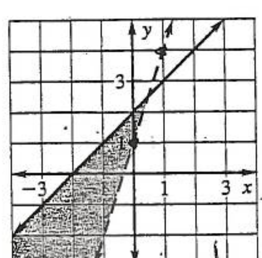
The system of inequalities for the shaded region is:

$y > \frac{3}{5}x - 2$	Inequality 1
$y \le -4x + 5$	Inequality 2

Exercises for Example 3

Write a system of inequalities that defines the shaded region.

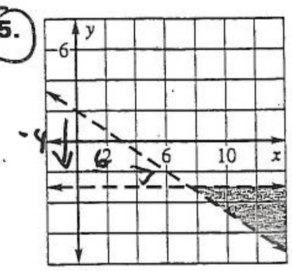
4.



$y > 3x + 1$

$y \le 1x + 2$

5.



$y < 3$

$y > -\frac{4}{6}x + 1$

$y > -\frac{2}{3}x + 1$