Advanced Algebra 2 **Ch. 12 Review Worksheet** **Name** Key

FROM SECTION 12.1

1) Write the first six terms of the sequence.

$a_n = n^2 + 6$

$a_1 = 7$ $a_4 = 22$
$a_2 = 10$ $a_5 = 31$
$a_3 = 15$ $a_6 = 42$

2) For the sequence, a. describe the pattern, b. write the next term, c. and write a rule for the nth term.

a) 2, 4, 8, 16 a. $\times 2$ geometric

b. 32

c. $a_n = 2 \cdot 2^{(n-1)}$

b) $\frac{4}{2}, \frac{8}{3}, \frac{12}{4}, \frac{16}{5}$ $+4$ $+1$ > arith.

b. $\frac{20}{6}$

c. top. $4+(n-1)4$ $4+4n-4$ $4n$ bottom $2+(n-1)1$ $n+1$

$\boxed{a_n = \frac{4n}{n+1}}$

3) Write the series using summation notation.

$-2 + 1 + 6 + 13 + 22 + \ldots$

+3 +5 +7 +9

1 1^2-3 -2
2 2^2-3 1
3 3^2-3 6
4 4^2-3 13 n^2-3

$$\boxed{\sum_{n=1}^{\infty} n^2 - 3}$$

4) Find the sum of the series.

$$\sum_{i=2}^{4} i^2 + i + 4$$

$a_2 = 10$
$a_3 = 16$
$a_4 = 24$

50

5) You want to save \$120 to buy an Ipod. You begin by saving 2 dollars in the first week. You plan to save an additional 3 dollars each week after that. For example, you will save \$5 the second week, \$8 in the third week, and so on. How many weeks must you save?

$a_1 = 2$
$d = 3$
$n =$ week #

$a_n = 2 + (n-1)3$
$a_n = 2 + 3n - 3$
$a_n = 3n - 1$

$120 = 3n - 1$
$121 = 3n$
$n = 40.3$
$\boxed{41 \text{ weeks}}$

FROM SECTION 12.2

6) Tell whether the sequence is arithmetic. Explain why or why not.

a) 2, -5, -12, -19, -26, … yes

constant difference

$d = -7$

b) 1, 2, 4, 7, 11, …

no

starts wit +1, then +2, then +3 etc

not a constant difference

7) Write a rule for the arithmetic sequence. Then find a_{10}.

a) -4, 2, 8, 14, 20 +6 $d = 6$

$a_n = -4 + (n-1)6$

$\boxed{a_n = 6n - 10}$

$\boxed{a_{10} = 50}$

b) $d = 2$, $a_6 = 10$

① $10 = a_1 + (6-1)2$

$10 = a_1 + 10$

$a_1 = 0$

$a_n = 0 + (n-1)2$

$\boxed{a_n = 2n - 2}$

$\boxed{a_{10} = 18}$

8) Write a rule for the nth term of the arithmetic sequence that has the two given terms.

$a_{20} = 240 \qquad a_{15} = 170$ need d, a_1

$240 = a_1 + (19)d$

$-(170 = a_1 + (14)d)$

$70 = 5d$

$d = 14$

$240 = a_1 + (19) \cdot 14$

$240 = a_1 + 266$

$a_1 = -26$

$a_n = -26 + (n-1)14$

$\boxed{a_n = 14n - 40}$

9) Find the sum of the arithmetic series. Use the formula.

$$\sum_{i=4}^{9} 6i - 30$$

$a_4 = 6 \cdot 4 - 30 = -6$

$6 \cdot 5 - 30 = 0$

$6 \cdot 6 - 30 = 6$

$6 \cdot 7 - 30 = 12$

$6 \cdot 8 - 30 = 18$

$a_9 = 6 \cdot 9 - 30 = 24$

Total sum $= 54$

$a_4 = -6$

$a_9 = 24$

$n = 6$

$S_6 = 6\left(\frac{-6+24}{2}\right) = 54$

10) An auditorium has 25 rows. The first row has 10 seats, and each row after the first has one more seat than the row before it. $a_1 = 10 \quad d = 1$

a) Write a rule for the nth row.

$a_n = 10 + (n-1)1$

$a_n = 9 + n$ or $n + 9$

b) There are 555 students in the school, can they all fit?

$S_n = n\left(\frac{a_1 + a_n}{2}\right) \qquad a_1 = 10 \quad a_{25} = 34$

$S_{25} = 25\left(\frac{10 + 34}{2}\right) = 550$ they can't all fit.

FROM SECTION 12.3

11) Tell whether the sequence is geometric. Explain why or why not.

a) $3, 5, 7, 9, 11, \ldots$

no, $\frac{5}{3} \neq \frac{7}{5}$

b) $5, 10, 20, 40, 80, \ldots$ $\times 2$

yes $\times 2$ each time $\frac{10}{5} = \frac{20}{10}$

12) Write a rule for the nth term of the geometric sequence. Find a_7. need a_1

$r = 3 \quad a_3 = 2$

$a_n = a_1 \cdot r^{n-1}$

$2 = a_1 \cdot 3^2$

$a_1 = 2/9$

$\boxed{a_n = \frac{2}{9} \cdot 3^{n-1}}$ $\boxed{a_7 = 162}$

13) Write a rule for the nth term of the geometric sequence that has two given terms.

$a_3 = 24 \, a_5 = 96$

$24 = a_1 \cdot r^2 \rightarrow a_1 = \frac{24}{r^2}$

$96 = a_1 \cdot r^4$

$96 = \frac{24}{r^2} \cdot r^4$

$96 = 24r^2$

$4 = r^2$

$r = 2$

$24 = a_1 \cdot 2^2$

$a_1 = 6$

$\boxed{a_n = 6(2)^{n-1}}$

14) Find the sum of the geometric series. Use the formula.

$$\sum_{i=1}^{10} 8(3)^{i-1}$$

n, r, a_1

$S_{10} = 8\left(\frac{1-3^{10}}{1-3}\right)$

$S_{10} = 8\left(\frac{-59048}{-2}\right) = \boxed{236192}$

15) You invest \$20,000 in a retirement plan. The plan is expected to have an annual return of 12%. Write a rule for the amount of money a_n available in the plan at the beginning of the nth year. What is the balance of the account at the beginning of the 20th year?

$a_1 = 20{,}000$

$r = 1.12$

$\boxed{a_n = 20{,}000(1.12)^{n-1}}$ $\boxed{a_{20} = \$172{,}255.23}$

FROM SECTION 12.4

16) Find the sum of the infinite geometric series, if it exists.

a) $\sum_{i=1}^{\infty} 5\left(\frac{1}{2}\right)^{i-1}$ yes

$S = \frac{5}{1-\frac{1}{2}}$

$S = 10$

b) $\sum_{i=0}^{\infty} 2(6)^{i-1}$

does not exist

$|r| > 1$

16) Find the sum of the infinite geometric series, if it exists.

c) $\frac{1}{2} + \frac{3}{4} + \frac{9}{8} + \frac{27}{16} + \frac{81}{32} + \ldots$ $r=\frac{3}{2}$

S= does not exist

d) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} + \ldots$ $r=\frac{1}{3}$

$S = \frac{2}{1-\frac{1}{3}} = \boxed{3}$

17) Write the repeating decimal as a reduced fraction. Show work.

a) .5555555… $a_1 = .5$ $r = .1$

$S = \frac{.5}{1-.1} = \frac{.5}{.9} = \boxed{\frac{5}{9}}$

b) $2.\overline{480}480480\ldots$

$x = 2.\overline{480}$

$1000x = 2480.\overline{480}$
$-x = 2.\overline{480}$
$999x = 2478$

$\boxed{x = \frac{2478}{999}}$

FROM SECTION 12.5

18) Write the first five terms of the sequence.

$a_1 = 3 \;\; a_n = 3a_{n-1} - 2$

$a_2 = 3 \cdot 3 - 2 = 7$
$a_3 = 3 \cdot 7 - 2 = 19$
$a_4 = 3 \cdot 19 - 2 = 55$
$a_5 = 3 \cdot 55 - 2 = 163$

$a_1 = 1, a_2 = 4 \;\; a_n = a_{n-1} - a_{n-2} + n$

$a_3 = 4 - 1 + 3 = 6$
$a_4 = 6 - 4 + 4 = 6$
$a_5 = 6 - 6 + 5 = 5$

19) Write a recursive rule for the sequence. The sequence may be arithmetic, geometric, or neither.

a) 2, 4, 6, 8, 10, … $+2$

$\boxed{a_1 = 2 \;\; a_2 = 4 \;\; a_n = a_{n-1} + 2}$

b) 2, 6, 18, 54, 162, … $\times 3$

$\boxed{a_1 = 2 \;\; a_2 = 6}$ $r=3$

$\boxed{a_n = 3 \cdot a_{n-1}}$

20) A chicken farm initially has 1000 chickens. Each year 20% of the chickens are killed and 120 new chickens are born.

a) Write a recursive rule for the number of chickens at the beginning of the nth year.

$a_1 = 100$
$r = .80$
add 120/yr.

$\boxed{a_n = .8 \cdot a_{n-1} + 120 \quad a_1 = 100 \;\; a_2 = 200}$

b) What value does the chicken population approach over time?

$a_1 = 100$
$a_2 = 200$
$a_3 = 280$
$a_4 = 344$
$a_5 = 395.2$
$a_6 = 436$
$a_7 = 468$
$a_8 = 495$
$a_9 = 516$
$a_{10} = 532$
$a_{11} = 546$
on calc.

approaches 600