Name ______________________ Date ______________

LESSON 8.1

Study Guide

For use with pages 506–513

GOAL **Find angle measures in polygons.**

Vocabulary

A **diagonal** of a polygon is a segment that joins two nonconsecutive vertices.

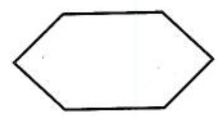
Theorem 8.1 Polygon Interior Angles Theorem: The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$.

Corollary to Theorem 8.1 Interior Angles of a Quadrilateral: The sum of the measures of the interior angles of a quadrilateral is 360°.

Theorem 8.2 Polygon Exterior Angles Theorem: The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

EXAMPLE 1 Find the sum of angle measures in a polygon

Find the sum of the measures of the interior angles of a convex hexagon.



Solution

A hexagon has 6 sides. Use the Polygon Interior Angles Theorem.

$(n - 2) \cdot 180° = (6 - 2) \cdot 180°$		Substitute 6 for n.
	$= 4 \cdot 180°$	Subtract.
	$= 720°$	Multiply.

The sum of the measures of the interior angles of a hexagon is 720°.

EXAMPLE 2 Find the number of sides of a polygon

The sum of the measures of the interior angles of a convex polygon is 2700°. Classify the polygon by the number of sides.

Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides n. Then solve the equation to find the number of sides.

$(n - 2) \cdot 180° = 2700°$	Polygon Interior Angles Theorem
$n - 2 = 15$	Divide each side by 180°.
$n = 17$	Add 2 to each side.

The polygon has 17 sides. It is a 17-gon.

LESSON 8.1

Name ______________________ Date ____________

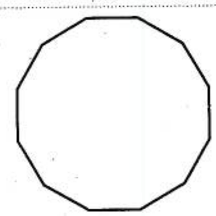
LESSON 8.1

Study Guide *continued*

For use with pages 506–513

Exercises for Examples 1 and 2

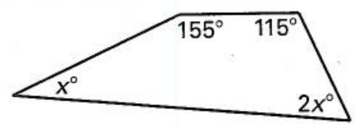
1. Find the sum of the measures of the interior angles of the polygon shown in the diagram. 1800°



2. The sum of the measures of the interior angles of a convex polygon is 540°. Classify the polygon by the number of sides. pentagon

EXAMPLE 3 Find an unknown interior angle measure

Find the value of x in the diagram.



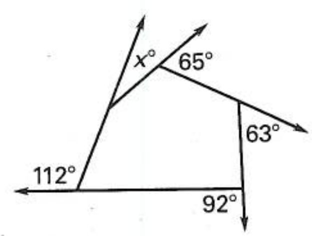
Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving x. Then solve the equation.

$x° + 2x° + 155° + 115° = 360°$	Corollary to Theorem 8.1
$3x + 270 = 360$	Combine like terms.
$x = 30$	Solve for x.

EXAMPLE 4 Find an unknown exterior angle measure

Find the value of x in the diagram.



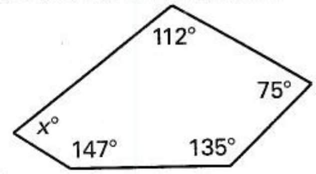
Solution

Use the Polygon Exterior Angles Theorem to write an equation involving x. Then solve the equation.

$x° + 65° + 63° + 92° + 112° = 360°$	Polygon Exterior Angles Theorem
$x + 332 = 360$	Combine like terms.
$x = 28$	Solve for x.

Exercises for Examples 3 and 4

3. What is the value of x in the diagram? 71°



4. A convex heptagon has exterior angles with measures 60°, 51°, 67°, 48°, 32°, and 59°. What is the measure of an exterior angle at the seventh vertex? 43°

Name ______________________________ Date ______________

LESSON 8.2

Study Guide

For use with pages 514–521

GOAL **Find angle and side measures in parallelograms.**

Vocabulary

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

Theorem 8.3: If a quadrilateral is a parallelogram, then its opposite sides are congruent.

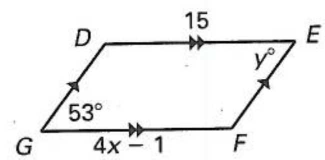
Theorem 8.4: If a quadrilateral is a parallelogram, then its opposite angles are congruent.

Theorem 8.5: If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

Theorem 8.6: If a quadrilateral is a parallelogram, then its diagonals bisect each other.

EXAMPLE 1 Use properties of parallelograms

Find the values of *x* and *y*.



Solution

DEFG is a parallelogram by the definition of a parallelogram. Use Theorem 8.3 to find the value of x.

$DE = FG$	Opposite sides of a $\square$ are $\cong$.
$15 = 4x - 1$	Substitute 15 for DE and $4x - 1$ for FG.
$4 = x$	Solve for x.

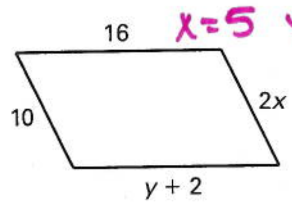
By Theorem 8.4, $\angle G \cong \angle E$, or $m\angle G = m\angle E$. So, $y° = 53°$.

In $\square DEFG$, $x = 4$ and $y = 53$.

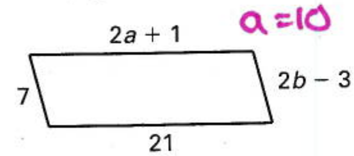
Exercises for Example 1

Find the value of each variable in the parallelogram.

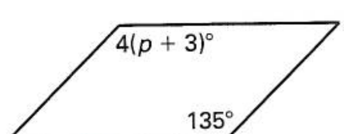
1. x=5 y=14



2. a=10 b=5

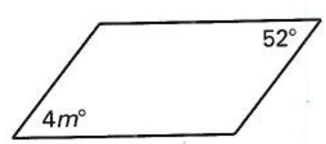


3.



p=33

4. m=13



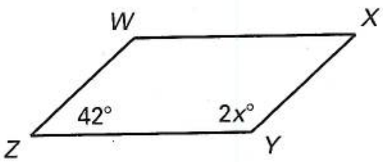
LESSON 8.2

Name ______________________ Date ______________

LESSON 8.2 **Study Guide** *continued*
For use with pages 514–521

EXAMPLE 2 Use properties of parallelograms

Find the value of x in $\square WXYZ$.



Solution

Use Theorem 8.5 to find the value of x.

$m\angle WZY + m\angle XYZ = 180^\circ$ Consecutive angles in a $\square$ are supplementary.

$42^\circ + 2x^\circ = 180^\circ$ Substitute.

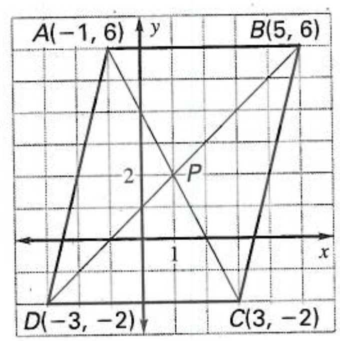
$x = 69$ Solve for x.

EXAMPLE 3 Find the intersection of diagonals

The vertices of $\square ABCD$ are $A(-1, 6)$, $B(5, 6)$, $C(3, -2)$, and $D(-3, -2)$. The diagonals of $\square ABCD$ intersect at point P. What are the coordinates of P?

Solution

STEP 1 Sketch $\square ABCD$ in the coordinate plane.



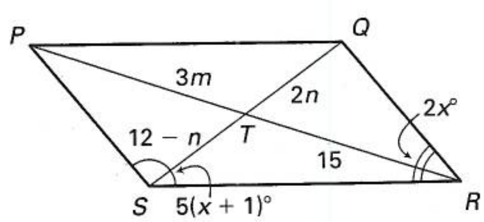
STEP 2 By Theorem 8.6, the diagonals of a parallelogram bisect each other. So, P is the midpoint of diagonals $\overline{AC}$ and $\overline{DB}$. Use the Midpoint Formula to find the midpoint P of $\overline{DB}$.

Midpoint: $\left(\frac{5 + (-3)}{2}, \frac{6 + (-2)}{2}\right) = (1, 2)$

The coordinates of P are (1, 2).

Exercises for Examples 2 and 3

Find the indicated measure in $\square PQRS$.



5. PR 30.75
6. ST 8
7. $m\angle SRQ$ 50°
8. $m\angle PQR$ 130°
9. The vertices of $\square ABCD$ are $A(-4, 2)$, $B(3, 2)$, $C(1, -1)$, and $D(-6, -1)$. The diagonals of $\square ABCD$ intersect at point P. What are the coordinates of P? $\left(-\frac{3}{2}, \frac{1}{2}\right)$
10. The vertices of $\square ABCD$ are $A(-5, 6)$, $B(1, 6)$, $C(4, 0)$, and $D(-2, 0)$. The diagonals of $\square ABCD$ intersect at point P. What are the coordinates of P? $\left(-\frac{1}{2}, 3\right)$

Name ______________________ Date ______________

LESSON 8.4

Study Guide

For use with pages 533–540

GOAL **Use properties of rhombuses, rectangles, and squares.**

Vocabulary

A **rhombus** is a parallelogram with four congruent sides.

A **rectangle** is a parallelogram with four right angles.

A **square** is a parallelogram with four congruent sides and four right angles.

Rhombus Corollary: A quadrilateral is a rhombus if and only if it has four congruent sides.

Rectangle Corollary: A quadrilateral is a rectangle if and only if it has four right angles.

Square Corollary: A quadrilateral is a square if and only if it is a rhombus and a rectangle.

Theorem 8.11: A parallelogram is a rhombus if and only if its diagonals are perpendicular.

Theorem 8.12: A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

Theorem 8.13: A parallelogram is a rectangle if and only if its diagonals are congruent.

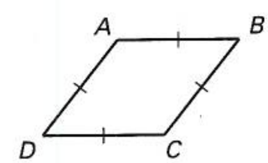
EXAMPLE 1 Use properties of special quadrilaterals

For any rhombus *ABCD*, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a sketch and explain your reasoning.

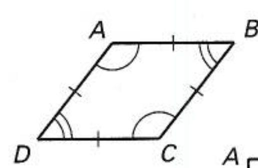
a. $AB > BC$ **b.** $m\angle A > m\angle B$

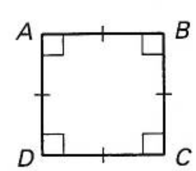
Solution

a. By definition, a rhombus is a parallelogram with four congruent sides. So, $AB = BC$. The statement is *never* true.



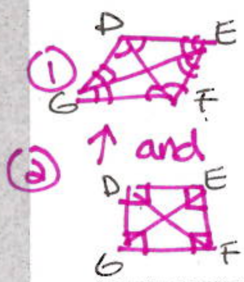
b. By definition, a rhombus is a parallelogram. By Theorem 8.4, opposite angles of a parallelogram are congruent. Because $\angle A$ and $\angle B$ are not opposite angles, they are not necessarily congruent, and $m\angle A$ could be greater than $m\angle B$. If rhombus *ABCD* is a square, then $m\angle A = m\angle B = 90°$. So, the statement is *sometimes* true.





Name ______________________________ Date ______________

LESSON **8.4** **Study Guide** *continued*
For use with pages 533–540



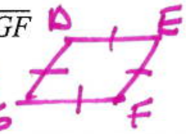
Exercises for Example 1

For any rhombus *DEFG*, decide whether the statement is *always*, *sometimes*, or *never* true. Draw a sketch and explain your reasoning.

1. $\angle DEG \cong \angle FEG$ always

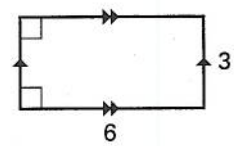
2. $\angle DEG \cong \angle EFD$ sometines

3. $\overline{DG} \cong \overline{GF}$ always



EXAMPLE 2 Classify special quadrilaterals

Classify the special quadrilateral. *Explain* your reasoning.



Solution

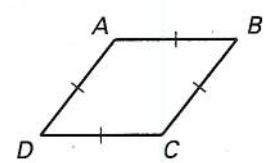
The quadrilateral is a parallelogram. By Theorem 8.4, opposite angles of a parallelogram are congruent, so all four angles of the quadrilateral are right angles. By the Rectangle Corollary, the quadrilateral is a rectangle. Because the four sides are not congruent, the rectangle is not a square.

EXAMPLE 3 List properties of special parallelograms

Sketch rhombus *ABCD*. List everything you know about it.

Solution

By definition, you need to draw a figure with the following properties:



- The figure is a parallelogram.
- The figure has four congruent sides.

Because *ABCD* is a parallelogram, it also has these properties:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

By Theorem 8.11, the diagonals of *ABCD* are also perpendicular.

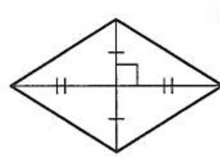
Exercises for Examples 2 and 3

Classify the special quadrilateral. *Explain* your reasoning.

4.

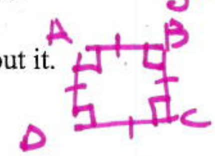
4 ≅ sides
rhombus
since all ∠'s
are ≅
360 ÷ 4 = 90°
so a SQUARE

5.



diagonals bisect
so parallelogram,
also ⊥ so
~~leg~~ rhombus

6. Sketch square *ABCD*. List everything you know about it.



see notes

Name ______________________ Date ____________

LESSON 8.5

Study Guide

For use with pages 542–549

GOAL **Use properties of trapezoids and kites.**

Vocabulary

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases.** For each of the bases of a trapezoid, there is a pair of **base angles,** which are the two angles that have that base as a side.

The nonparallel sides of a trapezoid are the **legs** of the trapezoid. If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid.** The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs.

A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

Theorem 8.14: If a trapezoid is isosceles, then each pair of base angles is congruent.

Theorem 8.15: If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

Theorem 8.16: A trapezoid is isosceles if and only if its diagonals are congruent.

Theorem 8.17 Midsegment Theorem for Trapezoids:
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

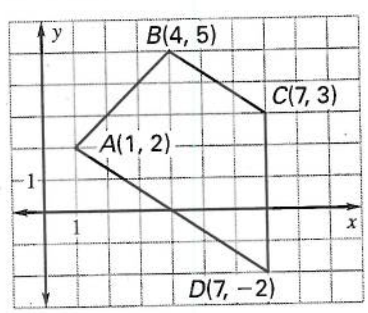
Theorem 8.18: If a quadrilateral is a kite, then its diagonals are perpendicular.

Theorem 8.19: If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

LESSON 8.5

EXAMPLE 1 **Use a coordinate plane**

Show that *ABCD* is a trapezoid.



Solution

Slope of $\overline{BC} = \frac{3-5}{7-4} = -\frac{2}{3}$

Slope of $\overline{AD} = \frac{-2-2}{7-1} = -\frac{2}{3}$

Slope of $\overline{AB} = \frac{5-2}{4-1} = 1$

Slope of $\overline{CD} = \frac{-2-3}{7-7} = \frac{-5}{0}$ Undefined

$\overline{BC}$ and $\overline{AD}$ have equal slopes, so they are parallel. $\overline{AB}$ and $\overline{CD}$ are not parallel. Because *ABCD* has exactly one pair of parallel sides, it is a trapezoid.

Name ______________________ Date ______________

LESSON 8.5

Study Guide *continued*

For use with pages 541–549

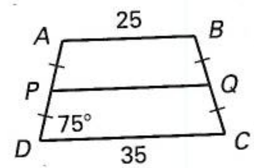
Exercise for Example 1

1. The vertices of $ABCD$ are $A(-5, 6)$, $B(1, 3)$, $C(0, 0)$, and $D(-7, 0)$. Show that $ABCD$ is a trapezoid.

one pair // sides
it is a trapezoid

Slope: $\overline{AB} = -\frac{1}{2}$ $\overline{DC} = 0$ $\overline{AD} = 3$ $\overline{BC} = 3$

EXAMPLE 2 Use properties of trapezoids



In the diagram, *ABCD* is an isosceles trapezoid and $\overline{PQ}$ is the midsegment.

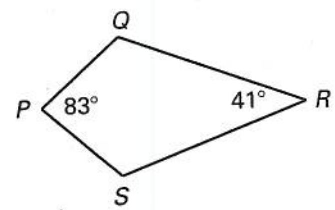
a. Find $m\angle B$. **b.** Find PQ.

Solution

a. Because $\angle D$ and $\angle A$ are consecutive interior angles formed by $\overleftrightarrow{AD}$ intersecting two parallel lines, they are supplementary. So, $m\angle A = 180° - 75° = 105°$. By Theorem 8.14, $\angle A \cong \angle B$. So, $m\angle B = 105°$.

b. By Theorem 8.17, $PQ = \frac{1}{2}(AB + CD) = \frac{1}{2}(25 + 35) = \frac{1}{2}(60) = 30$.

EXAMPLE 3 Use properties of kites



In the diagram, *PQRS* is a kite. Find $m\angle Q$.

Solution

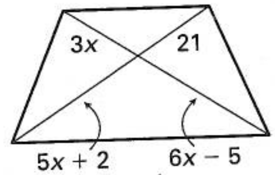
By Theorem 8.19, $PQRS$ has exactly one pair of congruent opposite angles. Because $\angle P \ncong \angle R$, $\angle Q$ and $\angle S$ must be congruent. So, $m\angle Q = m\angle S$.

$m\angle Q + m\angle S + 83° + 41° = 360°$	Corollary to Theorem 8.1
$m\angle Q + m\angle Q + 83° + 41° = 360°$	Substitute $m\angle Q$ for $m\angle S$.
$2(m\angle Q) + 124° = 360°$	Combine like terms.
$m\angle Q = 118°$	Solve for $m\angle Q$.

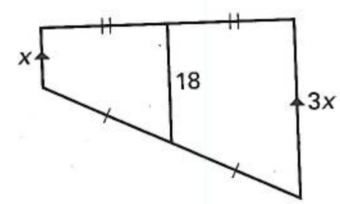
Exercises for Examples 2 and 3

Find the value of *x*.

2. 7



3. 9



4. In a kite, the measures of the angles are $6x°$, $24°$, $84°$, and $126°$. Find the value of x. What are the measures of the angles that are congruent? 21, 126°