Name ______________________ Date ______________

LESSON 5.1

Study Guide

For use with pages 294–301

GOAL **Use properties of midsegments and write coordinate proofs.**

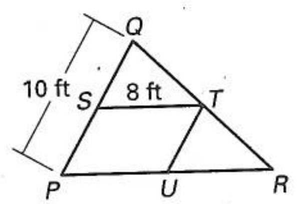
> **Vocabulary**
>
> A **midsegment of a triangle** is a segment that connects the midpoints of two sides of the triangle.
>
> A **coordinate proof** involves placing geometric figures in a coordinate plane. When you use variables to represent the coordinates of a figure in a coordinate proof, the results are true for all figures of that type.
>
> **Theorem 5.1 Midsegment Theorem:** The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

LESSON 5.1

EXAMPLE 1

Use the Midsegment Theorem to find lengths

In the diagram, $\overline{ST}$ and $\overline{TU}$ are midsegments of $\triangle PQR$. Find PR and TU.



Solution

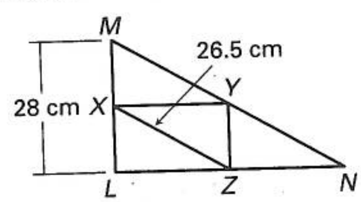
$PR = 2 \cdot ST = 2(8 \text{ ft}) = 16 \text{ ft}$

$TU = \frac{1}{2} \cdot QP = \frac{1}{2}(10 \text{ ft}) = 5 \text{ ft}$

Exercises for Example 1

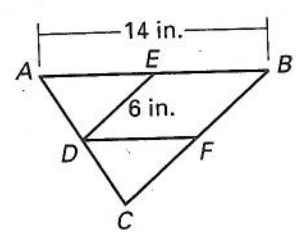
1. In the diagram, $\overline{XZ}$ and $\overline{ZY}$ are midsegments of $\triangle LMN$. Find MN and ZY.

MN = 53 cm
ZY = 14cm



2. In the diagram, $\overline{ED}$ and $\overline{DF}$ are midsegments of $\triangle ABC$. Find DF and BC.

DF = 7in.
BC = 12.in.

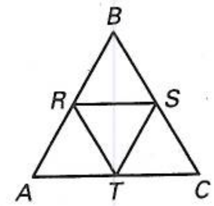


Name ____________________ Date ____________

LESSON 5.1 **Study Guide** *continued*
For use with pages 294–301

EXAMPLE 2 Use the Midsegment Theorem

In the diagram at the right, $\overline{SB} \cong \overline{SC}$, $\overline{RS} \parallel \overline{AC}$, and $RS = \frac{1}{2}AC$. Show that R is the midpoint of $\overline{BA}$.



Solution

Because $\overline{SB} \cong \overline{SC}$, S is the midpoint of $\overline{BC}$. Because $\overline{RS} \parallel \overline{AC}$ and $RS = \frac{1}{2}AC$, $\overline{RS}$ is a midsegment of $\triangle ABC$ by definition. By the Midsegment Theorem, R is the midpoint of $\overline{BA}$.

EXAMPLE 3 Place a figure in a coordinate plane

Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

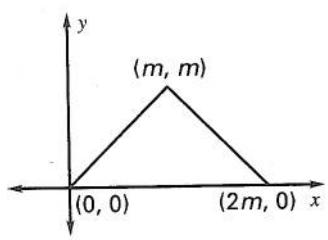
a. An isosceles triangle

b. A square

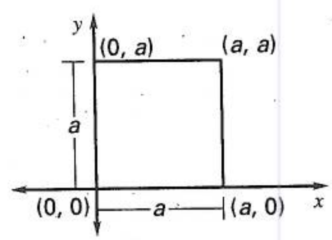
Solution

It is easy to find lengths of horizontal and vertical segments and distances from (0, 0), so place one vertex at the origin and one or more sides on an axis.

a. Let $2m$ represent the length of the base of the isosceles triangle. The coordinates of the vertex opposite the base is (m, m), which makes each of the legs congruent.



b. Let a represent the side length of the square.



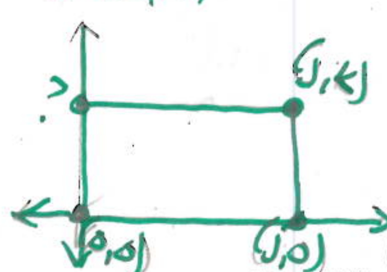
Exercises for Examples 2 and 3

3. In Example 2, if T is the midpoint of $\overline{AC}$, what do you know about $\overline{ST}$?

4. A rectangle has vertices (0, 0), $(j, 0)$, and (j, k). Find the fourth vertex.

#3. $\overline{ST} \parallel \overline{BA}$, $ST = \frac{1}{2}BA$

4. (0, K)



LESSON 5.1

Name ____________________ Date ____________________

Study Guide

For use with pages 303–309

GOAL **Use perpendicular bisectors to solve problems.**

Vocabulary

A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a **perpendicular bisector.**

A point is **equidistant** from two figures if the point is the *same distance* from each figure.

Theorem 5.2 Perpendicular Bisector Theorem: In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Theorem 5.3 Converse of the Perpendicular Bisector Theorem: In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

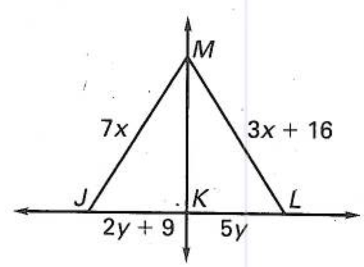
When three or more lines, rays, or segments intersect in the same point, they are called **concurrent** lines, rays, or segments. The point of intersection of the lines, rays, or segments is called the **point of concurrency.**

Theorem 5.4 Concurrency of Perpendicular Bisectors of a Triangle: The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

The point of concurrency of the three perpendicular bisectors of a triangle is called the **circumcenter** of the triangle.

EXAMPLE 1 Use the Perpendicular Bisector Theorem

$\overleftrightarrow{KM}$ is the perpendicular bisector of $\overline{JL}$. Find JK.

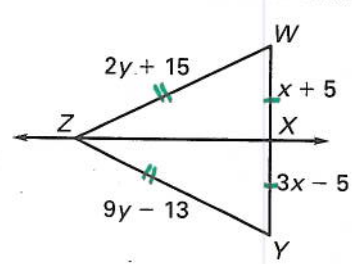


Solution

$JK = KL$	Perpendicular Bisector Theorem
$5y = 2y + 9$	Substitute.
$y = 3$	Solve for y.
$JK = 2(3) + 9 = 15$	

Exercises for Example 1

In the diagram $\overleftrightarrow{XZ}$, is the perpendicular bisector of $\overline{WY}$. $2y+15 = 9y-13$



1. Find WZ. $y=4$ $WZ=23$

2. Find XY. $x+5 = 3x-5$
$10=2x$ $XY=10$
$x=5$

Name _______________________________ Date _______________

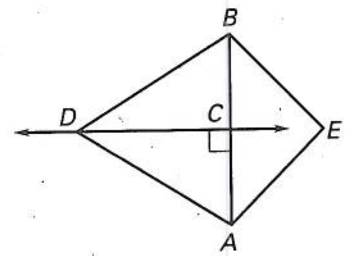
LESSON 5.2

Study Guide *continued*

For use with pages 303–309

EXAMPLE 2 Use perpendicular bisectors

In the diagram shown, $\overleftrightarrow{DC}$ is the perpendicular bisector of $\overline{AB}$ and $\overline{AE} \cong \overline{BE}$.



a. What segment lengths in the diagram are equal?

b. Is E on $\overleftrightarrow{DC}$?

Solution

a. $\overleftrightarrow{DC}$ bisects $\overline{AB}$, so $CA = CB$. Because D is on the perpendicular bisector of $\overline{AB}$, $DA = DB$ by Theorem 5.2. Because $\overline{AE} \cong \overline{BE}$, $AE = BE$ by definition of congruence.

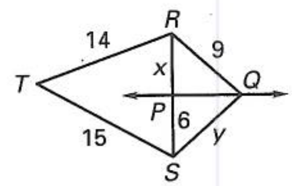
b. Because $AE = BE$, E is equidistant from A and B. So, by the Converse of the Perpendicular Bisector Theorem, E is on the perpendicular bisector of $\overline{AB}$, which is $\overleftrightarrow{DC}$.

(#3) $\overleftrightarrow{PQ}$ bisects $\overline{RS}$, so PR=PS. Because Q is on ⊥ bisector of $\overline{RS}$, QR=QS by ⊥ bisector th.

(#4) No, if T were on $\overleftrightarrow{PQ}$ then TR would be ≅ to ST T would be equidistant

Exercises for Example 2

In the diagram, $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{RS}$.

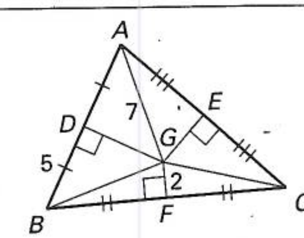


3. What segment lengths in the diagram are equal? *Explain* your reasoning.

4. Is T on $\overleftrightarrow{PQ}$? *Explain.*

EXAMPLE 3 Use the concurrency of perpendicular bisectors

The perpendicular bisectors of $\triangle ABC$ meet at point G. Find GB.



Solution

Using Theorem 5.4, you know that point G is equidistant from the vertices of the triangle. So, $GA = GB = GC$.

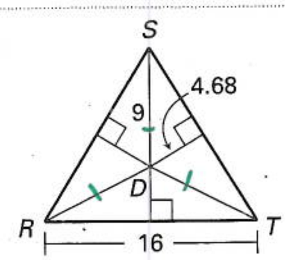
$GB = GA$ Theorem 5.4.

$GB = 7$ Substitute.

Exercise for Example 3

5. The perpendicular bisectos of $\triangle RST$ meet at point D. Find DR.

DR = 9



LESSON 5.2

Name ______________________ Date ______________

LESSON 5.3

Study Guide

For use with pages 310–316

GOAL **Use angle bisectors to find distance relationships.**

Vocabulary

The point of concurrency of the three angle bisectors of a triangle is called the **incenter** of the triangle.

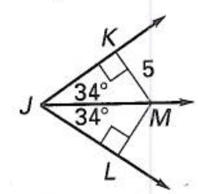
Theorem 5.5 Angle Bisector Theorem: If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

Theorem 5.6 Converse of the Angle Bisector Theorem: If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

Theorem 5.7 Concurrency of Angle Bisectors of a Triangle: The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

EXAMPLE 1 Use the Angle Bisector Theorem

Find the measure of $\overline{LM}$.

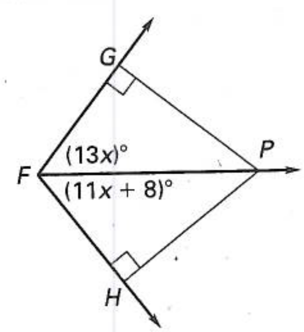


Solution

$\overrightarrow{JM}$ bisects $\angle KJL$ because $m\angle KJM = m\angle LJM$. Because $\overrightarrow{JM}$ bisects $\angle KJL$ and $\overline{MK} \perp \overline{JK}$ and $\overline{ML} \perp \overline{JL}$, $ML = MK$ by the Angle Bisector Theorem. So, $ML = MK = 5$.

EXAMPLE 2 Use algebra to solve a problem

For what value of x does P lie on the bisector of $\angle GFH$?



Solution

From the definition of an angle bisector, you know that P lies on the bisector of $\angle GFH$ if $m\angle GFP = m\angle HFP$.

$m\angle GFP = m\angle HFP$	Set angle measures equal.
$13x = 11x + 8$	Substitute.
$x = 4$	Solve for x.

Point P lies on the bisector of $\angle GFH$ when $x = 4$.

LESSON 5.3

Name ______________________ Date ______________

LESSON 5.3

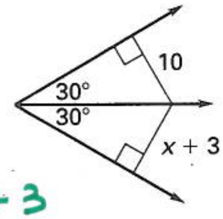
Study Guide *continued*

For use with pages 310–316

Exercises for Examples 1 and 2

Find the value of *x*.

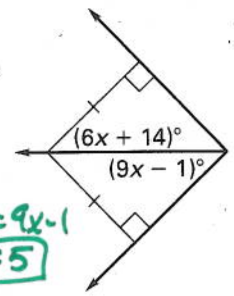
1.



$10 = x + 3$

$\boxed{x = 7}$

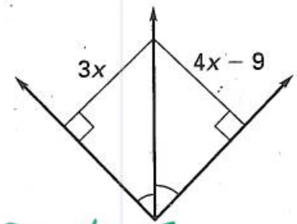
2.



$6x + 14 = 9x - 1$

$\boxed{x = 5}$

3.



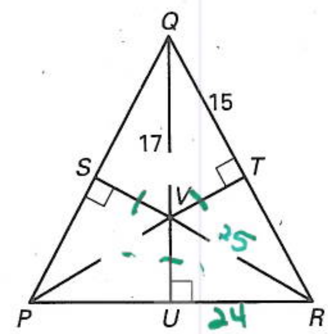
$3x = 4x - 9$

$\boxed{x = 9}$

EXAMPLE 3

Use the concurrency of angle bisectors

In the diagram, *V* is the incenter of $\triangle PQR$. Find *VS*.



Solution

By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter V is equidistant from the sides of $\triangle PQR$. So, to find VS, you can find VT in $\triangle PQR$ by using the Pythagorean Theorem.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$17^2 = VT^2 + 15^2$	Substitute known values.
$289 = VT^2 + 225$	Multiply.
$64 = VT^2$	Subtract 225 from each side.
$8 = VT$	Take the positive square root of each side.

Because $VT = VS$, $VS = 8$.

#4 $\overline{VU} \cong \overline{VS}$

$24^2 + VU^2 = 25^2$

$VU^2 = 49$

$VU = 7$

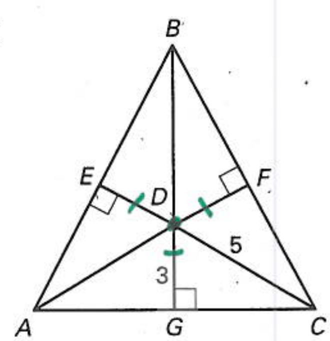
so $VS = 7$

Exercises for Example 3

4. In Example 3, suppose you are not given QV or QT, but you are given that $RU = 24$ and $RV = 25$. Find VS.

5. In the diagram, D is the incenter of $\triangle ABC$. Find DF.

#5 $DF = 3$



LESSON 5.3

Name ____________________ Date ____________

LESSON 5.4

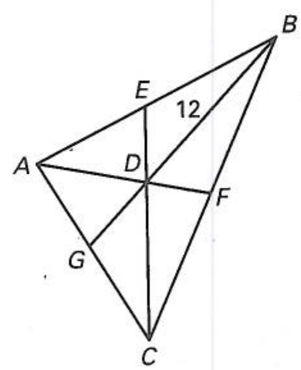
Study Guide
For use with pages 318–327

GOAL **Use medians and altitudes of triangles.**

> **Vocabulary**
>
> A **median of a triangle** is a segment from a vertex to the midpoint of the opposite side.
>
> The point of concurrency of the three medians of a triangle is called the **centroid,** and is always inside the triangle.
>
> An **altitude of a triangle** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.
>
> The point at which the lines containing the three altitudes of a triangle intersect is called the **orthocenter** of the triangle.
>
> **Theorem 5.8 Concurrency of Medians of a Triangle:** The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.
>
> **Theorem 5.9 Concurrency of Altitudes of a Triangle:** The lines containing the altitudes of a triangle are concurrent.

EXAMPLE 1 Use the centroid of a triangle

In $\triangle ABC$, D is the centroid and $BD = 12$. Find DG and BG. $\frac{2}{3} - \frac{1}{3}$



Solution

$BD = \frac{2}{3}BG$	Concurrency of Medians of a Triangle Theorem
$12 = \frac{2}{3}BG$	Substitute 12 for BD.
$18 = BG$	Multiply each side by the reciprocal, $\frac{3}{2}$.

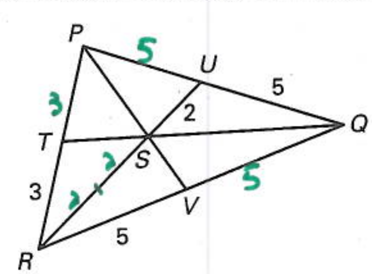
Then $DG = BG - BD = 18 - 12 = 6$.

So, $DG = 6$ and $BG = 18$.

Exercises for Example 1

$\frac{2}{3} \cdot \frac{1}{3}$

In $\triangle PQR$, S is the centroid, $\overline{PQ} \cong \overline{PQ}$, $UQ = 5$, $TR = 3$, and $SU = 2$.



1. Find RU and RS. RU = 6 RS = 4
2. Find the perimeter of $\triangle PQR$.

Perimeter = 26

Name ______________________ Date ____________

LESSON 5.4

Study Guide *continued*

For use with pages 318–327

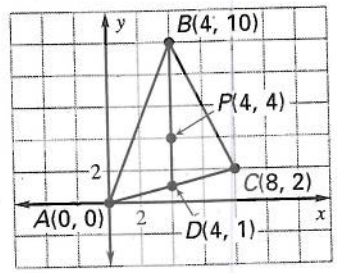
EXAMPLE 2 Find the centroid of a triangle

The vertices of $\triangle ABC$ are $A(0, 0)$, $B(4, 10)$, and $C(8, 2)$. Find the coordinates of the centroid P of $\triangle ABC$.

Solution

Sketch $\triangle ABC$. Then use the Midpoint Formula to find the midpoint D of $\overline{AC}$ and sketch median $\overline{BD}$.

$$D\left(\frac{0+8}{2}, \frac{0+2}{2}\right) = D(4, 1)$$



The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex $B(4, 10)$ to $D(4, 1)$ is $10 - 1 = 9$ units. So, the centroid is $\frac{2}{3}(9) = 6$ units down from B on $\overline{BD}$.

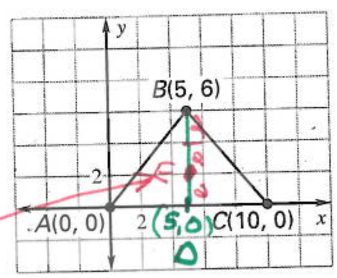
The coordinates of the centroid P are $(4, 10 - 6)$ or $(4, 4)$.

Exercises for Example 2

Find the coordinates of the centroid of the triangle with the given vertices.

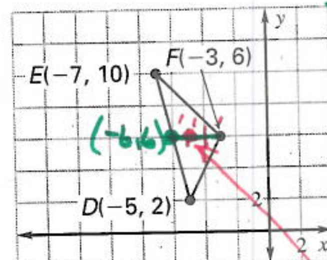
3.

BD = 6
E is centroid
2/3 − 1/3
so $\boxed{E(5,2)}$

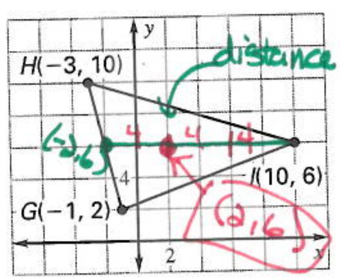


4.

#4
mdpt. ED $\left(\frac{-7+5}{2}, \frac{10+2}{2}\right)$
mdpt ED (−6,6)
distance btw mdpt. & F is 3
$\boxed{\text{centroid } (-5,6)}$



5.

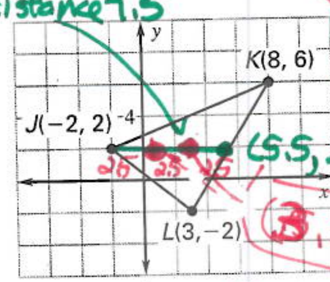


mdpt HG is $\left(\frac{-3+-1}{2}, \frac{10+2}{2}\right)$

(−2, 6)

6.

distance 7.5



mdpt. KL $\left(\frac{8+3}{2}, \frac{6+-2}{2}\right)$

(5.5, 2)

Name ______________________ Date ______________

LESSON 5.5

Study Guide

For use with pages 328–334

LESSON 5.5

GOAL **Find possible side lengths of a triangle.**

Vocabulary

Theorem 5.10: If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

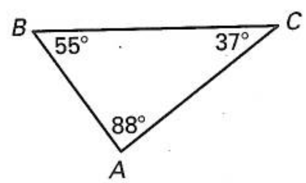
Theorem 5.11: If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller side.

Theorem 5.12 Triangle Inequality Theorem: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

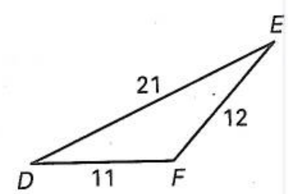
EXAMPLE 1 Write measurements in order from least to greatest

Write the measurements of the triangle in order from least to greatest.

a.



b.



Solution

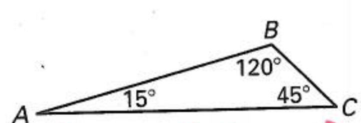
a. $m\angle C < m\angle B < m\angle A$
$AB < AC < BC$

b. $m\angle E < m\angle D < m\angle F$
$DF < EF < DE$

Exercises for Example 1

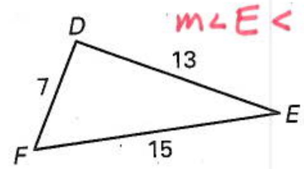
Write the measurements of the triangle in order from least to greatest.

1.

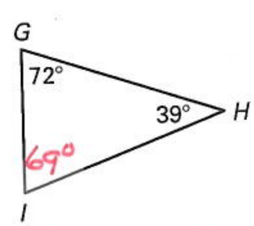


$m\angle A < m\angle C < m\angle B$

2. $m\angle E < m\angle F < m\angle D$

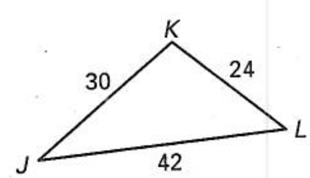


3.



$m\angle H < m\angle I < m\angle G$

4.



$m\angle J < m\angle L < m\angle K$

Study Guide *continued*

For use with pages 328–334

5. A triangle has sides that are about 33, 18, and 24 centimeters long and angles of about 32°, 103°, and 45°. Sketch and label a diagram with the shortest side on the bottom and the largest angle at the left.

6. A right triangle has sides that are 16, 34, and 30 inches long and angles of 90°, about 28°, and about 62°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the left.

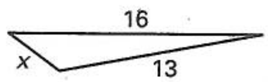
EXAMPLE 2 Find possible side lengths

A triangle has one side of length 13 and another side of length 16. Describe the possible lengths of the third side.

Solution

Let x represent the length of the third side. Draw diagrams to help visualize the small and large values of x. Then use the Triangle Inequality Theorem to write and solve the inequalities.

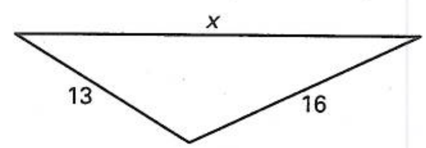
Small values of x



$x + 13 > 16$

$x > 3$

Large values of x



$13 + 16 > x$

$29 > x$, or $x < 29$

The length of the third side must be greater than 3 and less than 29.

Exercises for Example 2

Two sides of a triangle are given. Describe the possible lengths of the third side.

7. 2 centimeters and 5 centimeters

8. 7 inches and 12 inches

9. 4 feet and 10 feet

10. 11 meters and 10 meters

11. 9 inches and 25 inches

12. 1 mile and 8 miles

#7 $x+2>5$ $5+2>x$
$\boxed{3<x<7}$

#8 $x+7>12$ $7+12>x$
$x>5$ $19>x$
$\boxed{5<x<19}$

#9 $x+4>10$ $4+10>x$
$\boxed{6<x<14}$

#10 $x+10>11$ $10+11>x$
$\boxed{1<x<21}$

#11 $x+9>25$ $9+25>x$
$\boxed{16<x<34}$

#12 $x+1>8$ $1+8>x$
$\boxed{7<x<9}$