GOAL

Classify triangles and find measures of their angles.

Vocabulary

A triangle is a polygon with three sides.

A scalene triangle has no congruent sides.

An isosceles triangle has at least two congruent sides.

An equilateral triangle has three congruent sides.

An acute triangle has three acute angles.

A right triangle has one right angle.

An obtuse triangle has one obtuse angle.

An equiangular triangle has three congruent angles.

When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

Theorem 4.1 Triangle Sum Theorem: The sum of the measures of the interior angles of a triangle is 180°.

Theorem 4.2 Exterior Angle Theorem: The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Corollary to the Triangle Sum Theorem: The acute angles of a right triangle are complementary.

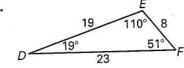
EXAMPLE 1

Copyright © by McDougal Littell, a division of Houghton Mifflin Company.

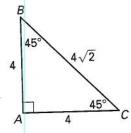
Classify triangles by sides and by angles

Classify the triangle by its sides and by its angles.

a.



b.



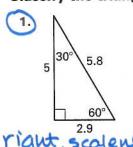
Solution

- **a.** Triangle DEF has one obtuse angle and no congruent sides. So, $\triangle DEF$ is an obtuse scalene triangle.
- **b.** Triangle ABC has one right angle and two congruent sides. So, $\triangle ABC$ is a right isosceles triangle.

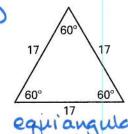
Study Guide continued For use with pages 216-224

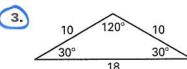
Exercises for Example 1

Classify the triangle by its sides and by its angles.



riant, scolene

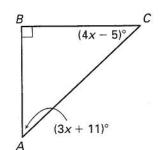




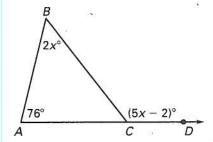
EXAMPLE 2

Find angle measures

a. Find $m \angle BAC$ and $m \angle BCA$.



b. Find $m \angle BCD$ and $m \angle ABC$.



Solution

a. $(4x-5)^{\circ} + (3x+11)^{\circ} = 90^{\circ}$

Use Corollary to the Triangle Sum Theorem.

$$x = 12$$

Solve for x.

So,
$$m \angle BCA = (4x - 5)^{\circ} = (4 \cdot 12 - 5)^{\circ} = 43^{\circ}$$
 and $m \angle BAC = (3x + 11)^{\circ} = (3 \cdot 12 + 11)^{\circ} = 47^{\circ}$.

b. $(5x-2)^{\circ} = 2x^{\circ} + 76^{\circ}$

Use Exterior Angle Theorem.

$$x = 26$$

Solve for x.

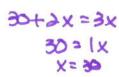
So,
$$m \angle BCD = (5x - 2)^{\circ} = (5 \cdot 26 - 2) = 128^{\circ}$$
 and $m \angle ABC = 2x^{\circ} = 2(26)^{\circ} = 52^{\circ}$.

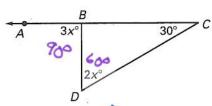
Exercises for Example 2

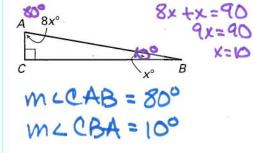




Find $m \angle CAB$ and $m \angle CBA$.







Copyright © by McDougal Littell, a division of Houghton Mifflin Company.

Study Guide

For use with pages 225-231

GOAL

Identify congruent figures.

Vocabulary

In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

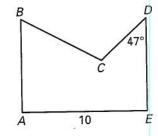
Theorem 4.3 Third Angles Theorem: If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.

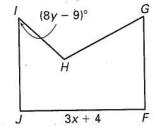
EXAMPLE 1

Use properties of congruent figures

In the diagram, $ABCDE \cong FGHIJ$.

- **a.** Find the value of x.
- **b.** Find the value of y.





Solution

a. You know that $\overline{AE} \cong \overline{FJ}$.

$$AE = JF$$

$$10 = 3x + 4$$

$$6 = 3x$$

$$2 = x$$

b. You know that $\angle D \cong \angle I$.

$$m \angle D = m \angle I$$

$$47^{\circ} = (8y - 9)^{\circ}$$

$$47 = 8y - 9$$

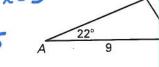
$$56 = 8y$$

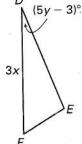
$$7 = y$$

Exercises for Example 1

In the diagram, $\triangle ABC \cong \triangle DEF$.

- 1. Find the value of x. $\times = 3$
- 2. Find the value of y.

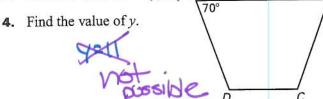


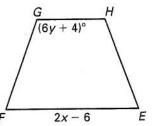


Study Guide continued For use with pages 225–231

In the diagram, $ABCD \cong EFGH$.

- 3. Find the value of x. \times

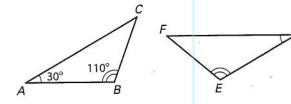




EXAMPLE 2

Use the Third Angles Theorem

Find $m \angle F$.

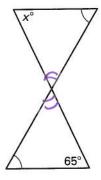


Solution

In the diagram, $\angle A \cong \angle D$ and $\angle B \cong \angle E$. So, by the Third Angles Theorem, $\angle C \cong \angle F$. By the Triangle Sum Theorem, $m \angle C = 180^{\circ} - 30^{\circ} - 110^{\circ} = 40^{\circ}$. So, $m \angle C = m \angle F = 40^{\circ}$ by the definition of congruent angles.

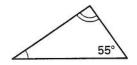
Exercises for Example 2

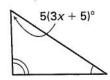
Find the value of x.



X=650

6.





Copyright © by McDougal Littell, a division of Houghton Mifflin Company.

Study Guide

For use with pages 233-239

GOAL

Use the side lengths to prove triangles are congruent.

Vocabulary

Postulate 19 Side-Side-Side (SSS) Congruence Postulate: If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

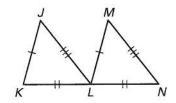
EXAMPLE 1

Use the SSS Congruence Postulate

Prove that $\triangle JKL \cong \triangle MLN$.

Solution

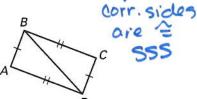
The marks on the diagram show that $\overline{JK} \cong \overline{ML}$, $\overline{KL} \cong \overline{LN}$, and $\overline{JL} \cong \overline{MN}$.



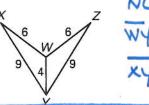
So, by the SSS Congruence Postulate, $\triangle JKL \cong \triangle MLN$.

Exercises for Example 1

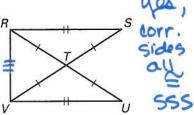
Decide whether the congruence statement is true. Explain your reasoning.



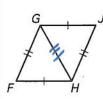
2. $\triangle XWY \cong \triangle WZY$



3. $\triangle RST \cong \triangle VUT$

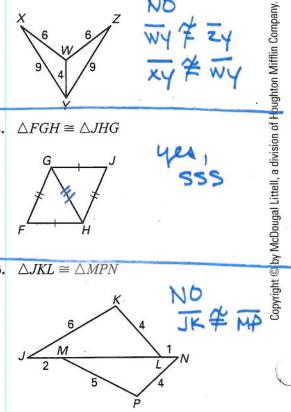


4. $\triangle FGH \cong \triangle JHG$



5. $\triangle PQR \cong \triangle RTS$

- $\triangle JKL \cong \triangle MPN$

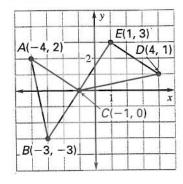


Study Guide continued

For use with pages 233-239

Congruent triangles in a coordinate plane

Use the SSS Congruence Postulate to show that $\triangle ABC \cong \triangle CDE$.



Solution

 $= \sqrt{13}$

Use the Distance Formula to show that corresponding sides are the same length.

$$AB = \sqrt{(-3 - (-4))^2 + (-3 - 2)^2}$$
$$= \sqrt{1^2 + (-5)^2}$$
$$= \sqrt{26}$$

$$CD = \sqrt{(4 - (-1))^2 + (1 - 0)^2}$$
$$= \sqrt{5^2 + 1^2}$$
$$= \sqrt{26}$$

So,
$$AB = CD$$
, and hence $\overline{AB} \cong \overline{CD}$.
 $BC = \sqrt{(-1 - (-3))^2 + (0 - (-3))^2}$
 $= \sqrt{2^2 + 3^2}$

$$DE = \sqrt{(1-4)^2 + (3-1)^2}$$
$$= \sqrt{(-3)^2 + 2^2}$$
$$= \sqrt{13}$$

So,
$$BC = DE$$
, and hence $\overline{BC} \cong \overline{DE}$.

$$CA = \sqrt{(-4 - (-1))^2 + (2 - 0)^2}$$
$$= \sqrt{(-3)^2 + 2^2}$$
$$= \sqrt{13}$$

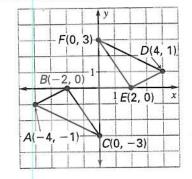
$$EC = \sqrt{(-1 - 1)^2 + (0 - 3)^2}$$
$$= \sqrt{(-2)^2 + (-3)^2}$$
$$= \sqrt{13}$$

So, CA = EC, and hence $\overline{CA} \cong \overline{EC}$.

So, by the SSS Congruence Postulate, you know that $\triangle ABC \cong \triangle CDE$.

Exercise for Example 2

7. Prove that $\triangle ABC \cong \triangle DEF$.



GOAL

Use sides and angles to prove congruence.

Vocabulary

In a right triangle, the sides adjacent to the right angle are called the **legs.**

The side opposite the right angle is called the **hypotenuse** of the right triangle.

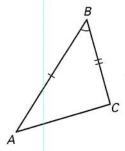
Postulate 20 Side-Angle-Side (SAS) Congruence Postulate: If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

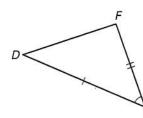
Theorem 4.5 Hypotenuse-Leg Congruence Theorem: If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

EXAMPLE 1

Use the SAS Congruence Postulate

Prove that $\triangle ABC \cong \triangle DEF$.





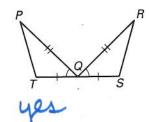
Solution

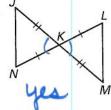
The marks on the diagram show that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\angle B \cong \angle E$. So, by the SAS Congruence Postulate, $\triangle ABC \cong \triangle DEF$.

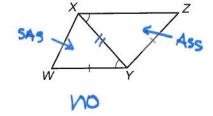
Exercises for Example 1

Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Postulate.

- **1.** $\triangle PQT$, $\triangle RQS$
- 2. △NKJ, △LKM
- **3.** $\triangle WXY$, $\triangle ZXY$





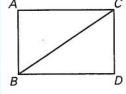


54

Write a proof.

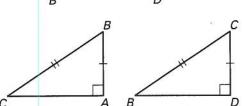
GIVEN: $\overline{AB} \cong \overline{DC}$, $\overline{BA} \perp \overline{AC}$, $\overline{CD} \perp \overline{DB}$

PROVE: $\triangle ABC \cong \triangle DCB$



Solution

Redraw the triangles so they are side by side with the corresponding parts in the same position. Mark the given information in the diagram.



Statements

- 1. $\overline{BA} \perp \overline{AC}, \overline{CD} \perp \overline{DB}$
- **2.** $\angle A$ and $\angle D$ are right angles.
- **3.** $\triangle ABC$ and $\triangle DCB$ are right triangles.
- **H** 4. $\overline{CB} \cong \overline{BC}$
- L 5. $\overline{AB} \cong \overline{DC}$
 - **6.** $\triangle ABC \cong \triangle DCB$

Reasons

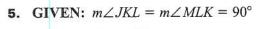
- 1. Given
- 2. Definition of ⊥ lines
- 3. Definition of a right triangle
- 4. Reflexive Property of Congruence
- 5. Given
- 6. HL Congruence Theorem

Exercises for Example 2

Write a proof.

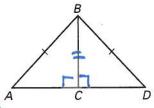
4. GIVEN: $\overline{AB} \cong \overline{DB}$, $\overline{BC} \perp \overline{AD}$

PROVE: $\triangle ABC \cong \triangle DBC$

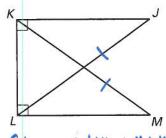


 $\overline{JL} \cong \overline{MK}$

PROVE: $\overline{JK} \cong \overline{ML}$



Given $\overline{AB} = \overline{DB}$, and $\overline{BC} \perp \overline{AD}$,
then $\angle ACB = \angle DCB$ because they
are both 90° by day, of perp.
lines. $\overline{CB} = \overline{CB}$ by the
neglexive property
therefore $\triangle ABC = \angle DBC$ by HL.



Given 2JKL=MMLK =90°, then A KLM and A LKM are rt. D's by def. of rt. D's.

Given JK = MK and we know

KL = KL by neglexive prop. Then

AKLM = ALKI by HO HL.

Chapter 4 Resource Book

55

Study Guide For use with pages 249–255

GOAL

Use two more methods to prove congruences.

Vocabulary

A flow proof uses arrows to show the flow of a logical argument.

Postulate 21 Angle-Side-Angle (ASA) Congruence Postulate:

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

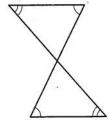
Theorem 4.6 Angle-Angle-Side (AAS) Congruence Theorem: If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

EXAMPLE 1

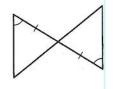
Identify congruent triangles

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

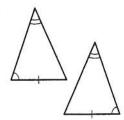
a.



b.



C.



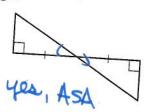
Solution

- a. The vertical angles are congruent, so three pairs of angles are congruent. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.
- **b.** The vertical angles are congruent, so two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.
- **c.** Two pairs of angles and a non-included pair of sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

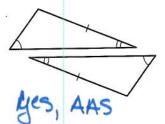
Study Guide continued For use with pages 249-255

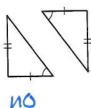
Exercises for Example 1

Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.



2.





EXAMPLE 2

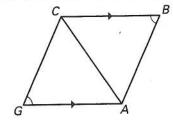
Write a flow proof

In the diagram, $\angle G \cong \angle B$ and $\overline{CB} || \overline{GA}$. Write a flow proof to show $\triangle GCA \cong \triangle BAC$.



GIVEN: $\angle G \cong \angle B$, $\overline{CB} || \overline{GA}$

PROVE: $\triangle GCA \cong \triangle BAC$



 $\overline{CB} \mid \overline{GA}$

 $\angle BCA \cong \angle GAC$

Given

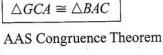
Alternate Interior &





 $\overline{AC} \cong \overline{AC}$

Reflexive Property



Exercises for Example 2

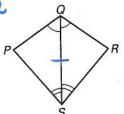
Write a flow proof to show that the triangles are congruent.

4. GIVEN: $\angle PQS \cong \angle RQS$

 $\angle QSP \cong \angle QSR$

PROVE: $\triangle PQS \cong \triangle RQS$

Givens 55 & 55 neglexine APOSE AROS



5. GIVEN: $\angle OMN \cong \angle ONM$

 $\angle LMO \cong \angle JNO$

PROVE: $\triangle MJN \cong \triangle NLM$

Copyright © by McDougal Littell, a division of Houghton Mifflin Company.

Study Guide

For use with pages 256-263

GOAL

Use congruent triangles to prove corresponding parts congruent.

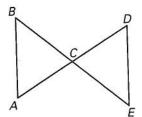
EXAMPLE 1

Identify congruent triangles

Explain how you can use the given information and congruent triangles to prove the statement.

GIVEN: $\overline{AB} \mid | \overline{DE}, \overline{AB} \cong \overline{DE}$

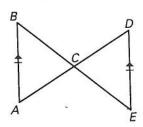
PROVE: C is the midpoint of \overline{BE} .



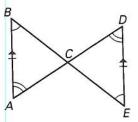
Solution

If you can show that $\triangle ABC \cong \triangle DEC$, you will know that C is the midpoint of \overline{BE} . First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle B \cong \angle E$ and $\angle A \cong \angle D$ by the Alternate Interior Angles Theorem.

Mark given information.



Add deduced information

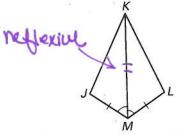


Two angle pairs and the included sides are congruent, so by the ASA Congruence Postulate, $\triangle ABC \cong \triangle DEC$. Because corresponding parts of congruent triangles are congruent, $\overline{BC} \cong \overline{CE}$. By the definition of midpoint, C is the midpoint of \overline{BE} .

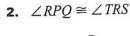
Exercises for Example 1

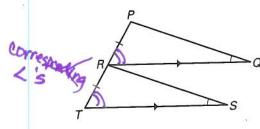
Tell which triangles you can show are congruent in order to prove the statement. What postulate or theorem would you use?

1.
$$\overline{JK} \cong \overline{LK}$$



SAS - then CACTC





AAS - then CACTO

and Litter II a division of Houghton Mifflin Company.

Study Guide continued For use with pages 256-263

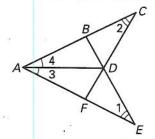
EXAMPLE 2

Plan a proof involving pairs of triangles

Use the given information to write a plan for a proof.

GIVEN: $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$

PROVE: $\triangle DEF \cong \triangle DCB$



Solution

In $\triangle DEF$ and $\triangle DCB$, you know $\angle 1 \cong \angle 2$. If you can show that $\angle EDF \cong \angle CDB$ and $\overline{ED} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

Because $\angle EDF$ and $\angle CDB$ are vertical angles, $\angle EDF \cong \angle CDB$ by the Vertical Angles Theorem.

To prove that $\overline{ED} \cong \overline{CD}$, you can first prive that $\triangle AED \cong \triangle ACD$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{AD} \cong \overline{AD}$ by the Reflexive Property. You can use the ASA Congruence Postulate to prove that $\triangle AED \cong \triangle ACD$.

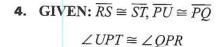
Plan for Proof: Use the ASA Congruence Postulate to prove that $\triangle AED \cong \triangle ACD$. Then state that $\overline{DE} \cong \overline{DC}$ because corresponding parts of congruent triangles are congruent. Use the ASA Congruence Postulate to prove that $\triangle DEF \cong \triangle DCB$.

Exercises for Example 2

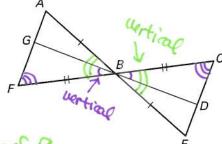
Use the diagram and the given information to write a plan for a proof.

3. GIVEN: $\overline{AB} \cong \overline{EB}, \overline{FB} \cong \overline{CB}$

PROVE: $\overline{BG} \cong \overline{BD}$

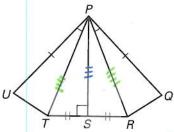


PROVE: $\triangle PTU \cong \triangle PRO$



MOW LAFB & LECB by CACTC LGBFIL DBC by westical

BG = BO by CACTC



Chapter 4 Resource Book

Study Guide For use with pages 264–270

GOAL

Use theorems about isosceles and equilateral triangles.

Vocabulary

When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.

Theorem 4.7 Base Angles Theorem: If two sides of a triangle are congruent, then the angles opposite them are congruent.

Theorem 4.8 Converse of Base Angles Theorem: If two angles of a triangle are congruent, then the sides opposite them are congruent.

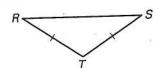
Corollary to the Base Angles Theorem: If a triangle is equilateral, then it is equiangular.

Corollary to the Converse of Base Angles Theorem: If a triangle is equiangular, then it is equilateral.

EXAMPLE 1

Identify congruent angles

In the diagram, $\overline{RT}\cong \overline{ST}$. Name two congruent angles.



Solution

 $\overline{RT} \cong \overline{ST}$, so by the Base Angles Theorem, $\angle R \cong \angle S$.

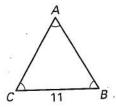
EXAMPLE 2

Find measures in a triangle

Find AB and AC in the triangle at the right.

Solution

The diagram shows that $\triangle ABC$ is equiangular. Therefore, by the Corollary to the Converse of Base Angles Theorem, $\triangle ABC$ is equilateral. So, AB = BC = AC = 11.



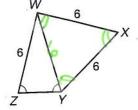
11:4011 a division of Houghton Mifflin Company

Exercises for Examples 1 and 2

Use the information in the diagram to find the given values.

- 1. Find WY. wy = 6
- **2.** Find $m \angle WXY$.





Study Guide continued For use with pages 264–270

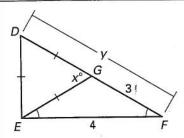
EXAMPLE 3

Use isosceles and equilateral triangles

In the diagram, $m\angle DEF = 90^{\circ}$. Find the values of x and y.

Solution

STEP 1 Find the value of x. Because $\triangle DEG$ is equilateral, it is also equiangular, and $m \angle GDE = m \angle DEG = x^{\circ}$. So, by the Triangle Sum Theorem, $3x^{\circ} = 180^{\circ}$, and x = 60.



STEP 2 Find the value of y. Because $\angle GEF \cong \angle GFE$, $\overline{GE} \cong \overline{GF}$ by the Converse of Base Angles Theorem, so GE = 3. Because $\triangle DEG$ is equilateral, DE = DG = GE = 3. Because $m \angle DEF = 90^{\circ}$, $\triangle DEF$ is a right triangle. Using the Pythagorean Theorem, $y = \sqrt{3^2 + 4^2} = 5$.

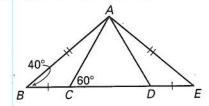
EXAMPLE 4

Copyright © by McDougal Littell, a division of Houghton Mifflin Company.

Solve a multi-step problem

Use the diagram to answer the questions.

a. What congruence postulate can you use to prove that $\triangle ABC \cong \triangle AED$?



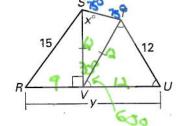
- **b.** Explain why $\triangle ACD$ is equiangular.
- **c.** Show that $\triangle ABD \cong \triangle AEC$.

Solution

- **a.** You can see that $\overline{AB} \cong \overline{AE}$ and $\overline{BC} \cong \overline{ED}$. By the Base Angles Theorem, you know that $\angle B \cong \angle E$. So, by the SAS Congruence Postulate, $\triangle ABC \cong \triangle AED$.
- **b.** Because corresponding parts of congruent triangles are congruent, you know that $\angle ACB \cong \angle ADE$, and by the Congruent Supplements Theorem, $\angle ACD \cong \angle ADC$. So $m\angle ADC = m\angle ACD = 60^{\circ}$, and $m\angle CAD = 180^{\circ} 60^{\circ} 60^{\circ} = 60^{\circ}$, and $\triangle ACD$ is equiangular.
- **c.** From part (b) you know that $\triangle ACD$ is equiangular. So, $\angle ADB \cong \angle ACE$ and therefore $\triangle ABD \cong \triangle AEC$ by the AAS Congruence Postulate.

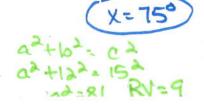
Exercises for Examples 3 and 4

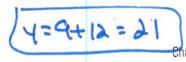
3. Find the values of x and y in the diagram at the right.



4. In Example 4 above, show that $\triangle ABD \cong \triangle AEC$ using the SSS Congruence Postulate.

1-60-90= 38





Geometry Chapter 4 Resource Book